[Print]

Assignment 15

Due: 11:00pm on Friday, December 4, 2009

Note: To understand how points are awarded, read your instructor's Grading Policy.

[Return to Standard Assignment View]

Electricity and Water Analogy

Learning Goal: To understand the analogy between water pressure, water flow, voltage, and current

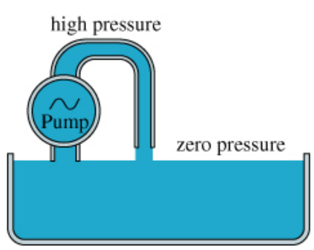
As suggested by the fact that we call both *currents*, the flow of charged particles through an electrical circuit is analogous in some ways to the flow of water through a pipe.

When water flows from a small pipe to a large pipe, the flow (measured, for instance, in gallons per minute) is the same in both pipes, because the amount of water entering one pipe must equal the amount leaving the other. If not, water would accumulate in the pipes. For the same reason, the total electric current I is constant for circuit elements in series.

Water pressure is analogous to total electric potential (voltage), and a pump is analogous to a battery. Water flowing through pipes loses pressure, just as current flowing through a resistor falls to lower voltage. A pump uses mechanical work to raise the water's pressure and thus its potential energy; in a battery, chemical reactions cause charges to flow against the average local electric field, from low to high voltage, increasing their potential energy.

Part A

Consider the following water circuit: water is continually pumped to high pressure by a pump, and then funnelled into a pipe that has lower pressure at its far end (else the water would not flow through the pipe) and back to the pump. Two such circuits are identical, except for one difference: the pipes in one circuit have a larger diameter than the pipes in the other circuit. Through which circuit is the flow of water greater?



ANSWER:

- Small pipe
- Large pipe (selected)

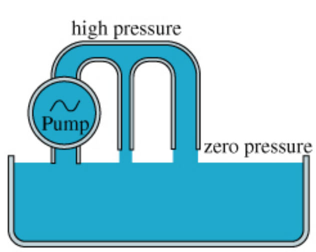
Correct

The cross sectional area of the pipe is analogous to the area of a wire: the smaller the area the higher the resistance and the more the pipe/wire impedes flow. If the change in pressure (proportional to the potential energy per unit mass) of water traveling through two pipes is the same, the flow will be less through the pipe with smaller cross sectional area. The electrical analog is Ohm's law $I = \frac{V}{R}$, where resistance R is inversely proportional to the area of a wire/resistor.

Part B

Now consider a variant on the circuit. The water is pumped to high pressure, but the water then faces a fork in the pipe. Two pipes lead back to the pump: large pipe L and small pipe S. Since the water can flow through either pipe, the pipes are said to be *in parallel*:

The overall flow of water that enters the system before the fork is equal to _____



Hint B.1 **Water conservation**

Hint not displayed

ANSWER:

- the flow through pipe L.
- the sum of the flows through L and S. (selected)
- the average of the flows through L and S.

Correct

Part C

What can you say about the drop in potential energy (per unit mass or volume) of water traveling through either pipe?

ANSWER:

- ○ The drop is greater for pipe L.
- ○ The drop is greater for pipe S.
- ◉ The drop is the same for both pipes.

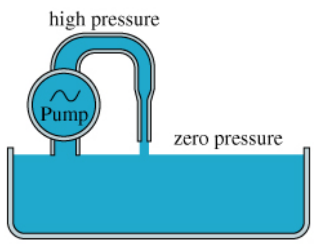
Correct

The pressure is equal at the top of both pipes (to the pressure created by the pump) and at the bottom of both pipes (taken to be zero). So the pressure drop across each pipe is equal; only the flows are different. This circuit is the analog of resistors in parallel, where the voltage is the same for both resistors, but the currents differ if the resistances are unequal.

Part D

Consider a new circuit: water is pumped to high pressure and fed into only one pipe. The pipe has two distinct segments of different diameters; the second half of the pipe has a smaller diameter than the first half:

Which of the following statements about the flow and change in pressure through each segment is true?



ANSWER:

- ○ The flow through each segment is the same as the overall flow; the change in pressure through each segment is the same as the overall change.
- ◉ The flow through each segment is the same as the overall flow; the sum of the changes in pressure through each segment equals the overall change.
- ○ The sum of the flows through each segment equals the overall flow; the change in pressure through each segment is the same as the overall change.
- ○ The sum of the flows through each segment equals the overall flow; the sum of the changes in pressure through each segment equals the overall change.

Correct

This circuit is the analog of resistors in series. Each segment of pipe has the same amount of water flowing through it, but a different pressure change. The overall resistance of the pipe is the sum of the resistances of each segment: $R_{\rm net} = R_1 + R_2$.

Tactics Box 32.1 Using Kirchhoff's Loop Law

Learning Goal: To practice Tactics Box 32.1 Using Kirchhoff's Loop Law.

Circuit analysis is based on Kirchhoff's laws, which can be summarized as follows:

- *Kirchhoff's junction law* says that the total current into a junction must equal the total current leaving the junction.
- *Kirchhoff's loop law* says that if we add all of the potential differences around the loop formed by a circuit, the sum of these potential differences must be zero.

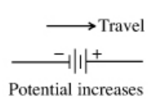
Although Kirchhoff's junction law is needed only when there are one or more junctions in a circuit, Kirchhoff's loop law is used for analyzing any type of circuit, as explained in the following tactics box.

TACTICS BOX 32.1 Using Kirchhoff's loop law

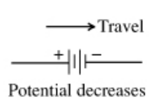
1. **Draw a circuit diagram.** Label all known and unknown quantities.
2. **Assign a direction to the current.** Draw and label a current arrow I to show your choice.
   - If you know the actual current direction, choose that direction.
   - If you don't know the actual current direction, make an educated guess. All that will happen if you choose wrong is that your value for I will end up negative.
3. **"Travel" around the loop.** Start at any point in the circuit; then, go all the way around the loop in the direction you assigned to the current in Step 2. As you go through each circuit element, ΔV is interpreted to mean

$$\Delta V = V_{\rm downstream} - V_{\rm upstream}.$$

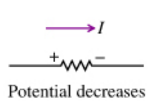
   - For an ideal battery with current in the negative-to-positive direction: $\Delta V_{\rm bat} = +\mathcal{E}$.



   - For an ideal battery in the positive-to-negative direction (i.e., the current is going into the positive terminal of the battery): $\Delta V_{\rm bat} = -\mathcal{E}$.



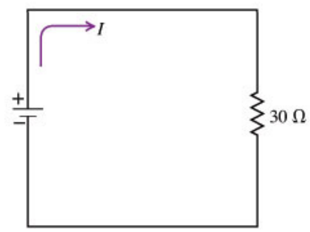
   - For a resistor: $\Delta V_{\rm R} = -IR$.



4. **Apply the loop law**: $\Sigma(\Delta V)_i = 0$.

Part A

The current in the circuit shown in the figure is 0.20 A. What is the potential difference $\Delta V_{\rm bat}$ across the battery traveling in the direction shown in the figure?



Hint A.1 **Find the potential difference across the resistor**

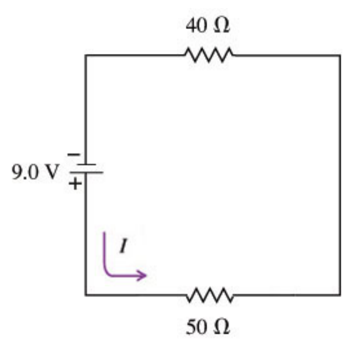
Hint not displayed

Express your answer in volts.

ANSWER: $\Delta V_{\rm bat}$ = **6.0** V
Correct

Part B

Find the current I in the circuit shown in the figure.



Hint B.1 **Find the potential difference across the battery**

Hint not displayed

Hint B.2 **Find the potential difference across the 40-ohm resistor**

Hint not displayed

Hint B.3 **Find the potential difference across the 50-ohm resistor**

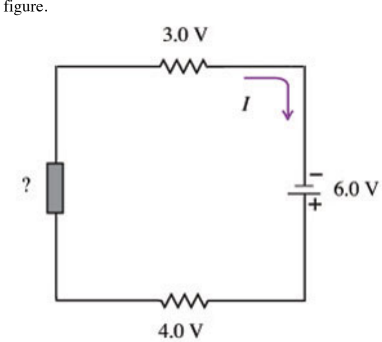
Hint not displayed

Express your answer in amperes.

ANSWER: I = **0.10** A
Correct

Part C

What is the potential difference ΔV across the unknown element in the circuit shown in the figure? Express your answer as if traveling across the element in the direction shown in the figure.



Hint C.1 **Apply Kirchhoff's loop law**

Hint not displayed

Express your answer in volts.

ANSWER: ΔV = **1.0** V
Correct

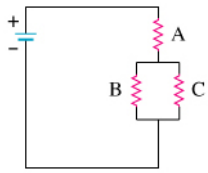
Kirchhoff's Current Rule Ranking Task

The placement of resistors in a circuit is one factor that can determine the current passing through the resistor. You will be given three circuits, and for each circuit you will be asked to compare the current through the various resistors.

In each of the circuits in Parts A to C, all resistors are identical.

Part A

Rank the resistors in the figure below (A to C) on the basis of the current that flows through them.

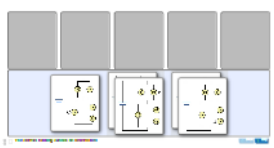


Hint A.1 **Kirchhoff's current rule for circuit junctions**

Hint not displayed

Rank from largest to smallest. To rank items as equivalent, overlap them.

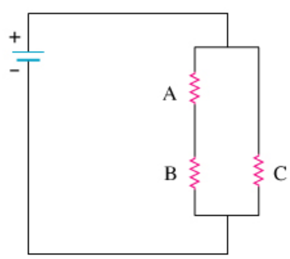
ANSWER:



View
Correct

Part B

Rank the resistors in the figure below (A to C) on the basis of the current that flows through them.



Hint B.1 **Kirchhoff's current rule for circuit junctions**

Hint not displayed

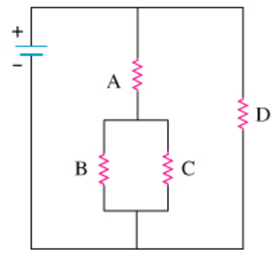
Rank from largest to smallest. To rank items as equivalent, overlap them.

ANSWER:

View
Correct

Part C

Rank the resistors in the figure below (A to D) on the basis of the current that flows through them.



Hint C.1 **Kirchhoff's current rule for circuit junctions**

Hint not displayed

Rank from largest to smallest. To rank items as equivalent, overlap them.

ANSWER:

View

Correct

Brightness of Light Bulbs Ranking Task

Part A

Consider a circuit containing five identical light bulbs and an ideal battery. Assume that the resistance of each light bulb remains constant. Rank the bulbs (A through E) based on their brightness.

Hint A.1 **How to approach the problem**

Hint not displayed

Hint A.2 **Comparing bulb A to bulb B**

Hint not displayed

Hint A.3 **Comparing bulb D to bulb E**

Hint not displayed

Hint A.4 **Comparing bulb C to bulb D or E**

Hint not displayed

Hint A.5 **Comparing bulb C to bulb A or B**

Hint not displayed

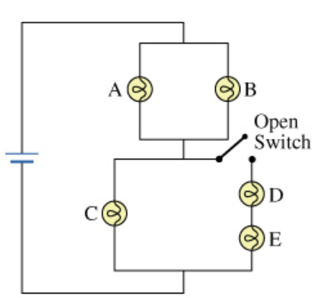
Rank from brightest to dimmest. To rank items as equivalent, overlap them.

ANSWER:

View

Correct

Now consider what happens when a switch in the circuit is opened.



Part B

What happens to the brightness of bulb A?

Hint B.1 **How to approach this part**

Hint not displayed

Hint B.2 **Consider changes in resistance**

Hint not displayed

ANSWER:

- (selected) It gets dimmer.
- It gets brighter.
- There is no change.

Correct

Part C

What happens to bulb C?

Hint C.1 **How to approach this part**

Hint not displayed

Hint C.2 **Find the current in bulb C earlier**

Hint not displayed

Hint C.3 **Find the current in bulb C now**

Hint not displayed

ANSWER:

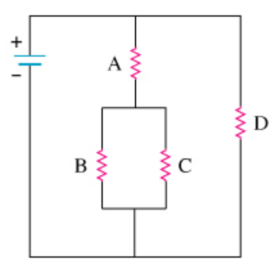
- It gets dimmer.
- (selected) It gets brighter.
- There is no change.

Correct

This is why appliances in your home are always connected in parallel. Otherwise, turning some of them on or off would cause the current in others to change, which could damage them.

Kirchhoff's Loop Rule Conceptual Question

The circuit shown belowconsists of four different resistors and a battery. You don't know the strength of the battery or the value any of the four resistances.



Part A

Select the expressions that will be equal to the voltage of the battery in the circuit, where $V_{\rm A}$, for example, is the potential drop across resistor A.

Hint A.1 **Kirchhoff's voltage rule for closed circuit loops**

Hint not displayed

Check all that apply.

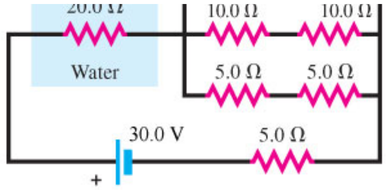
ANSWER:

- [x] $V_{\rm A}+V_{\rm B}$
- [x] $V_{\rm A}+V_{\rm C}$
- [ ] $V_{\rm B}+V_{\rm C}$
- [ ] $V_{\rm A}+V_{\rm B}+V_{\rm C}$
- [ ] $V_{\rm A}+V_{\rm B}+V_{\rm C}+V_{\rm D}$
- [ ] $V_{\rm A}+V_{\rm D}$
- [x] $V_{\rm D}$

Correct

Heating a Water Bath

In the circuit in the figure, a 20-ohm resistor sits inside 115 $\rm g$ of pure water that is surrounded by insulating Styrofoam.



Part A

If the water is initially at temperature 10.5°C, how long will it take for its temperature to rise to 58.1°C?

Hint A.1 **How to approach the problem**

Hint not displayed

Hint A.2 **Calculate the resistance of the circuit**

Hint not displayed

Hint A.3 **Calculate the current in the equivalent resistor**

Hint not displayed

Hint A.4 **Calculate the current through the resistor in the water bath**

Hint not displayed

Hint A.5 **Calculate the power output of the resistor**

Hint not displayed

Hint A.6 **Heating the water**

Hint not displayed

Use $4190 \ \mathrm{J/kg \cdot C^{\circ}}$ as the heat capacity of water, and express your answer in seconds using three significant figures.

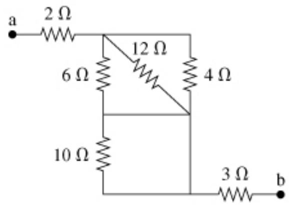
ANSWER: t = **1150** s
Correct

Problem 32.42

Part A

What is the equivalent resistance between points $\mathbf{a}$ and $\mathbf{b}$ in the figure?

Express your answer using one significant figure.



ANSWER: R_{ab} = **7** Ω
Correct

Problem 32.9

A standard 100 W (120 V) lightbulb contains a 7.60 cm-long tungsten filament. The high-temperature resistivity of tungsten is $9.0 \times 10^{-7} \Omega \ \mathrm{m}$.

Part A

What is the diameter of the filament?

ANSWER: **24.6** $\mu\mathrm{m}$
Correct

Problem 32.38

You've made the finals of the Science Olympics! As one of your tasks, you're given 2.2 g of aluminum and asked to make a wire, using all the aluminum that will dissipate 9.0 W when connected to a 2.9 V battery.

Part A

What length will you choose for your wire?

Express your answer using two significant figures.

ANSWER: L = **5.2** m
Correct

Part B

What diameter will you choose for your wire?

Express your answer using two significant figures.

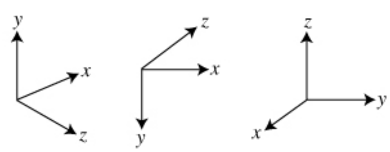
ANSWER: d = **0.45** mm
Correct

Force on Moving Charges in a Magnetic Field

Learning Goal: To understand the force on a charge moving in a magnetic field.

Magnets exert forces on other magnets even though they are separated by some distance. Usually the force on a magnet (or piece of magnetized matter) is pictured as the interaction of that magnet with the *magnetic field* at its location (the field being generated by other magnets or currents). More fundamentally, the force arises from the interaction of individual moving charges within a magnet with the local magnetic field. This force is written $\vec{F} = q\vec{v} \times \vec{B}$, where $\vec{F}$ is the force, q is the individual charge (which can be negative), $\vec{v}$ is its velocity, and $\vec{B}$ is the local magnetic field.

This force is nonintuitive, as it involves the vector product (or cross product) of the vectors $\vec{v}$ and $\vec{B}$. In the following questions we assume that the coordinate system being used has the conventional arrangement of the axes, such that it satisfies $\hat{x} \times \hat{y} = \hat{z}$, where $\hat{x}$, $\hat{y}$, and $\hat{z}$ are the unit vectors along the respective axes.



Three right-handed coordinate systems

Let's go through the right-hand rule. Starting with the generic vector cross-product equation $\vec{A} = \vec{B} \times \vec{C}$ point your forefinger of your right hand in the direction of $\vec{B}$, and point your middle finger in the direction of $\vec{C}$. Your thumb will then be pointing in the direction of $\vec{A}$.

Part A

Consider the specific example of a positive charge q moving in the +*x* direction with the local magnetic field in the +*y* direction. In which direction is the magnetic force acting on the particle?

Express your answer using unit vectors (e.g., $\hat{x}$- $\hat{y}$). (Recall that $\hat{x}$ is written *x_unit*.)

ANSWER: Direction of $\vec{F}_{\rm mag}$ = $\hat{z}$
Correct

Part B

Now consider the example of a positive charge q moving in the +*x* direction with the local magnetic field in the +*z* direction. In which direction is the magnetic force acting on the particle?

Express your answer using unit vectors.

ANSWER: Direction of $\vec{F}_{\rm mag}$ = $-\hat{y}$
Correct

Part C

Now consider the example of a positive charge q moving in the *xy* plane with velocity $\vec{v} = v\cos(\theta)\hat{x} + v\sin(\theta)\hat{y}$ (i.e., with magnitude v at angle θ with respect to the *x* axis). If the local magnetic field is in the +*z* direction, what is the direction of the magnetic force acting on the particle?

Hint C.1 **Finding the cross product**

Hint not displayed

Express the direction of the force in terms of θ, as a linear combination of unit vectors, $\hat{x}$, $\hat{y}$, and $\hat{z}$.

ANSWER: Direction of $\vec{F}_{\mathrm{mag}}$ = $\sin(\theta)\,\hat{x} - \cos(\theta)\,\hat{y}$

Correct

Part D

First find the magnitude of the force F on a positive charge q in the case that the velocity $\vec{v}$ (of magnitude v) and the magnetic field $\vec{B}$ (of magnitude B) are perpendicular.

Express your answer in terms of v, q, B, and other quantities given in the problem statement.

ANSWER: F = $|q|\,vB$

Correct

Part E

Now consider the example of a positive charge q moving in the -z direction with speed v with the local magnetic field of magnitude B in the +z direction. Find F, the magnitude of the magnetic force acting on the particle.

Express your answer in terms of v, q, B, and other quantities given in the problem statement.

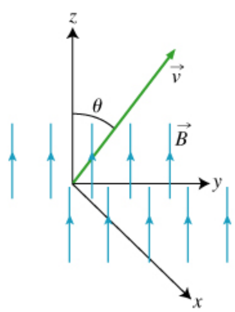
ANSWER: F = **0**

Correct

There is no magnetic force on a charge moving parallel or antiparallel to the magnetic field. Equivalently, the magnetic force is proportional to the component of velocity perpendicular to the magnetic field.

Part F

Now consider the case in which the positive charge q is moving in the yz plane with a speed v at an angle θ with the z axis as shown (with the magnetic field still in the +z direction with magnitude B). Find the magnetic force $\vec{F}$ on the charge.



Hint F.1 **Direction of force**

Hint not displayed

Hint F.2 **Relevant component of velocity**

Hint not displayed

Express the magnetic force in terms of given variables like q, v, B, θ, and unit vectors.

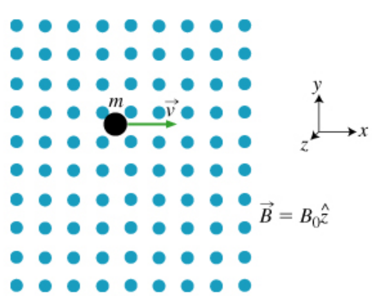
ANSWER: $\vec{F}$ = $qvB\sin(\theta)\,\hat{x}$

Correct

Charge Moving in a Cyclotron Orbit

Learning Goal: To understand why charged particles move in circles perpendicular to a magnetic field and why the frequency is an invariant.

A particle of charge q and mass m moves in a region of space where there is a uniform magnetic field $\vec{B} = B_0\hat{z}$ (i.e., a magnetic field of magnitude B_0 in the +z direction). In this problem, neglect any forces on the particle other than the magnetic force.



Part A

At a given moment the particle is moving in the $+x$ direction (and the magnetic field is always in the $+z$ direction). If q is positive, what is the direction of the force on the particle due to the magnetic field?

Hint A.1 **The right-hand rule for magnetic force**

Hint not displayed

ANSWER:

- $+x$ direction
- $-x$ direction
- $+y$ direction
- $-y$ direction
- $+z$ direction
- $-z$ direction

Correct

Part B

This force will cause the path of the particle to curve. Therefore, at a later time, the direction of the force will ___________.

ANSWER:

- have a component along the direction of motion
- remain perpendicular to the direction of motion
- have a component against the direction of motion
- first have a component along the direction of motion; then against it; then along it; etc.

Correct

Part C

The fact that the magnetic field generates a force perpendicular to the instantaneous velocity of the particle has implications for the work that the field does on the particle. As a consequence, if only the magnetic field acts on the particle, its kinetic energy will ___________.

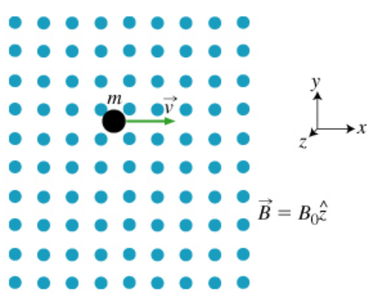
ANSWER:

- increase over time
- decrease over time
- remain constant
- oscillate

Correct

Part D

The particle moves in a plane perpendicular to the magnetic field direction as shown in the figure. What is ω, the angular frequency of the circular motion?



Hint D.1 **How to approach the problem**

Hint not displayed

Hint D.2 **Determine the magnetic force**

Hint not displayed

Hint D.3 **Determine the acceleration of the particle**

Hint not displayed

Hint D.4 **Express the angular speed in terms of the linear speed**

Hint not displayed

Express ω in terms of q, m, and B_0.

ANSWER:

$\omega = \frac{qB_0}{m}$

Correct

Note that this result for the frequency does not depend on the radius R of the circle. Although it appeared in the equations of force and motion, it canceled out. This implies that the frequency (but not the linear speed) of the particle is invariant with orbit size.

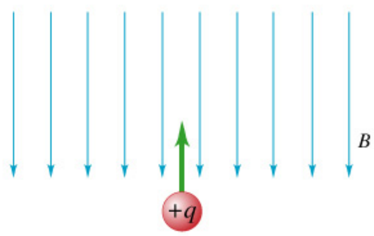
The first particle accelerator built, the *cyclotron*, was based on the fact that the frequency of a charged particle orbiting in a uniform field is independent of the radius. In the cyclotron, radio frequency voltage is applied across a gap between the two sides of the conducting vacuum chamber in which the protons circulate owing to an external magnetic field. Particles in phase with this voltage are accelerated each time they cross the gap (because the field reverses while they make half a circle) and reach energies of millions of electron volts after several thousand round trips.

Magnetic Force on Charged Particles Conceptual Question

For each of the situations below, a charged particle enters a region of uniform magnetic field. Determine the direction of the force on each charge due to the magnetic field.

Part A

Determine the direction of the force on the charge due to the magnetic field.



Hint A.1 Determining the direction of a magnetic force

Hint not displayed

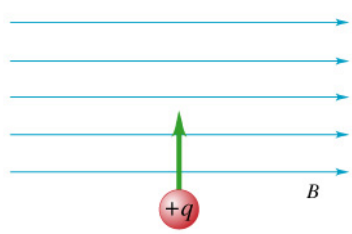
ANSWER:

- ○ $\vec{F}$ points into the page.
- ○ $\vec{F}$ points out of the page.
- ○ $\vec{F}$ points neither into nor out of the page and $\vec{F} \neq 0$.
- ⊙ $\vec{F} = 0$.

Correct

Part B

Determine the direction of the force on the charge due to the magnetic field.



Hint B.1 Determining the direction of a magnetic force

Hint not displayed

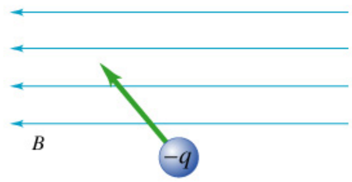
ANSWER:

- ○ $\vec{F}$ points out of the page.
- ⊙ $\vec{F}$ points into the page.
- ○ $\vec{F}$points neither into nor out of the page and $\vec{F} \neq 0$.
- ○ $\vec{F} = 0$.

Correct

Part C

Determine the direction of the force on the charge due to the magnetic field. Note that the charge is negative.



Hint C.1 **Effect of a magnetic field on a negative charge**

Hint not displayed

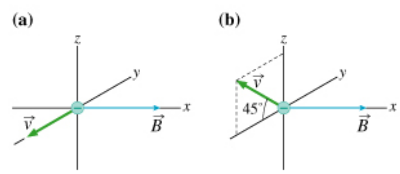
ANSWER:

- ○ $\vec{F}$ points out of the page.
- ◉ $\vec{F}$ points into the page.
- ○ $\vec{F}$ points neither into nor out of the page and $\vec{F} \neq 0$.
- ○ $\vec{F} = 0$.

Correct

Problem 33.27

An electron moves in the magnetic field $\vec{B}$= 0.540 $\hat{i}$ T with a speed of 1.20 $\times\ 10^7\ \mathrm{m/s}$ in the directions shown in the figure. For each, what is magnetic force $\vec{F}$ on the electron?



Part A

Express vector $\vec{F}$ in the form of F_x, F_y, F_z, where the x, y, and z components are separated by commas.

ANSWER: $\vec{F}$ = **0,0,−1.04×10^{-12}** N

Correct

Part B

Express vector $\vec{F}$ in the form of F_x, F_y, F_z, where the x, y, and z components are separated by commas.

ANSWER: $\vec{F}$ = **0,−7.33×10^{-13},−7.33×10^{-13}** N

Correct

Problem 33.33

The Hall voltage across a 1.00-mm-thick conductor in a 0.900 T magnetic field is $3.2\mu\mathrm{V}$ when the current is 18.0 A.

Part A

What is the charge-carrier density in this conductor?

ANSWER: **3.16×10^{28}** m^{-3}

Correct

Score Summary:

Your score on this assignment is 100.3%.
You received 14.04 out of a possible total of 14 points.