28.1. Solve: The wire's cross-sectional area is \( A = \pi r^2 = \pi (1.0 \times 10^{-3} \text{ m})^2 = 3.1415 \times 10^{-6} \text{ m}^2 \). and the electron current through this wire is \( 2.0 \times 10^{19} \text{ s}^{-1} \). Using Table 28.1 for the electron density of iron and Equation 28.3, the drift velocity is

\[
v_d = \frac{i}{nA} = \frac{2.0 \times 10^{19} \text{ s}^{-1}}{(8.5 \times 10^{28} \text{ m}^{-3})(3.1415 \times 10^{-6} \text{ m}^2)} = 7.5 \times 10^{-5} \text{ m/s} = 75 \text{ pm/s}
\]

Assess: The drift speed of electrons in metals is small.

28.2. Solve: We estimate a distance of 5 ft from the wall switch to the ceiling and then a distance of 8 ft to the center of the room. This yields a total length of approximately \( L = 4.0 \text{ m} \) that an electron will travel in time \( t \). The drift speed of electrons in copper is \( v_d = 1.0 \times 10^{-4} \text{ m/s} \). Thus,

\[
t = \frac{L}{v_d} = \frac{4.0 \text{ m}}{1.0 \times 10^{-4} \text{ m/s}} = 4.0 \times 10^4 \text{ s} \approx 11 \text{ hours}
\]

Assess: Given a drift speed of 100 pm/s, a time of 11 hours to cover 4.0 m is physically reasonable.

28.3. Solve: Using Equation 28.3 and Table 28.1, the electron current is

\[
i = nAv_d = (5.9 \times 10^{28} \text{ m}^{-3}) \pi (0.5 \times 10^{-3} \text{ m}) (5.0 \times 10^{-4} \text{ m/s}) = 2.3 \times 10^{14} \text{ s}^{-1}
\]

The time for 1 mole of electrons to pass through a cross section of the wire is

\[
t = \frac{N_e \times 1 \text{ mole}}{i} = \frac{6.02 \times 10^{23}}{2.3 \times 10^{14} \text{ s}^{-1}} = 2.62 \times 10^5 \text{ s} \approx 3.03 \text{ days}
\]

Assess: The drift speed is small, and Avogadro’s number is large. A time of the order of 3 days is reasonable.

28.4. Solve: Equation 28.2 is \( N_e = nAv_d \Delta t \). Using Table 28.1 for the electron density, we get

\[
A = \frac{\pi D^2}{4} = \frac{N_e}{nv_d \Delta t}
\]

\[
\Rightarrow D = \sqrt{\frac{4N_e}{\pi nv_d \Delta t}} = \sqrt{\frac{4(1.0 \times 10^{16})}{\pi (5.8 \times 10^{28} \text{ m}^{-3})(8.0 \times 10^{-4} \text{ m/s})(320 \times 10^{-6} \text{ s})}} = 9.26 \times 10^{-4} \text{ m} = 0.926 \text{ mm}
\]

28.5. Solve: Using Equation 28.2,

\[
n = \frac{N_e}{Av_d \Delta t} = \frac{1.44 \times 10^{14}}{(4.0 \times 10^{-6} \text{ m}^2)(2.0 \times 10^{-4} \text{ m/s})(3.0 \times 10^{-6} \text{ s})} = 6.0 \times 10^{26} \text{ m}^{-3}
\]

From Table 28.1, the metal is aluminum.

28.6. Solve: For \( L = 1.0 \text{ cm} = 1.0 \times 10^{-2} \text{ m} \), the surface area of the wire is

\[
A = (2\pi r)L = \pi DL = \pi (1.0 \times 10^{-3} \text{ m})(1.0 \times 10^{-2} \text{ m}) = (1.0 \times 10^{-5} \text{ m}^2)\pi
\]
The surface charge density of the wire is
\[ \eta = \frac{Q}{A} = \frac{(1000 \text{ cm}^{-1} \times 1 \text{ cm})1.60 \times 10^{-19} \text{ C}}{(1.0 \times 10^{-3} \text{ m})^2} = 5.1 \times 10^{-12} \text{ C/m}^2 \]

28.7. Solve: (a) Each gold atom has one conduction electron. Using Avogadro's number and \( n \) as the number of moles, the number of atoms is
\[ N = nN_A = \frac{m}{M_A}N_A = \frac{\rho V}{M_A}N_A = \frac{\rho \pi^2 L}{M_A}N_A \]
The density of gold is \( \rho = 19,300 \text{ kg/m}^3 \), the atomic mass is \( M_A = 197 \text{ g mol}^{-1} \), \( r = 0.5 \times 10^{-3} \text{ m} \), \( L = 0.1 \text{ m} \), and \( N_A = 6.02 \times 10^{23} \text{ mol}^{-1} \). Substituting these values, we get \( N = 4.63 \times 10^{21} \) electrons.
(b) If all the electrons are transferred a charge of \((4.63 \times 10^{21})(-1.60 \times 10^{-19} \text{ C}) = -740.8 \text{ C} \) will be delivered. To deliver a charge of \(-32 \text{ nC} \), however, the electrons within a length \( l \) have to be delivered. Thus,
\[ l = \frac{-32 \times 10^{-22} \text{ C}}{-740.8 \text{ C}} \times 10 \text{ cm} = 4.32 \times 10^{-11} \text{ cm} = 4.32 \times 10^{-12} \text{ m} \]

28.8. Model: Use the conduction model to relate the drift speed to the electric field strength.
Solve: From Equation 28.7, the electric field is
\[ E = \frac{mv_d}{e\tau} = \frac{(9.11 \times 10^{-31} \text{ kg})(2.0 \times 10^{-4} \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(5.0 \times 10^{-14} \text{ s})} = 0.023 \text{ N/C} \]

28.9. Model: We will use the model of conduction to relate the electric field strength to the mean free time between collisions.
Solve: From Equation 28.8, the electric field is
\[ E = \frac{mi}{ne\tau^2} = \frac{(9.11 \times 10^{-31} \text{ kg})(5.0 \times 10^{19} \text{ s}^{-1})}{(8.5 \times 10^{28} \text{ m}^{-1})(1.60 \times 10^{-19} \text{ C})(4.2 \times 10^{-15} \text{ s})\pi(0.9 \times 10^{-5} \text{ m})^2} = 0.31 \text{ N/C} \]

28.10. Visualize:

Solve: (a) The charge density is not uniform along the wire. If it was, there would be no electric field inside the wire. The charge density is most positive near the positive terminal of the battery. It gradually decreases until becoming neutral halfway around the circuit. It then becomes increasingly negative, and is most negative near the negative terminal of the battery.
(b) The electric field values are shown in the figure.

28.11. Model: A battery is a charge escalator.
Solve: When a wire is connected to a battery, there is a sustained motion of electrons. A current of 1.5 A means that a charge of 1.5 C flows through a cross section of the wire per second. Because \( Q = Ne \), the number of electrons transported per second is
\[ N_e = \frac{Q}{e} = \frac{1.5 \text{ C}}{1.60 \times 10^{-19} \text{ C}} = 9.4 \times 10^{18} \text{ electrons} \]

28.12. Solve: Equation 28.10 is \( Q = I\Delta t \). The amount of charge delivered is
\[ Q = (10.0 \text{ A})(5.0 \text{ min} \times \frac{60 \text{ s}}{1 \text{ min}}) = 3000 \text{ C} \]
The number of electrons that flow through the hair dryer is
\[ N = \frac{Q}{e} = \frac{3000 \text{ C}}{1.60 \times 10^{-19} \text{ C}} = 1.88 \times 10^{22} \]
28.13. Solve: From Equation 28.10,
\[ I = \frac{Q}{\Delta t} = \frac{Ne}{\Delta t} = \frac{(2.0 \times 10^{15})(1.60 \times 10^{-19} \text{ C})}{1.0 \times 10^{-3} \text{ s}} = 0.0032 \text{ C/s} = 0.0032 \text{ A} = 3.2 \text{ mA} \]

28.14. Visualize:

The direction of the current \( I \) in a material is opposite to the direction of motion of the negative charges and is the same as the direction of motion of positive charges.

Solve: The charge due to positive ions moving to the right per second is

\[ q_+ = N_+ (2e) = (5.0 \times 10^{15}) (2 \times 1.60 \times 10^{-19} \text{ C}) = 1.60 \times 10^{-3} \text{ C} \]

The charge due to negative ions moving to the left per second is

\[ q_- = N_- (-e) = (6.0 \times 10^{15}) (-1.60 \times 10^{-19} \text{ C}) = -0.96 \times 10^{-3} \text{ C} \]

Thus, the current in the solution is

\[ i = \frac{q_+ - q_-}{t} = \frac{1.60 \times 10^{-3} \text{ C} - (-0.96 \times 10^{-3} \text{ C})}{1 \text{ s}} = 2.56 \times 10^{-3} \text{ A} = 2.56 \text{ mA} \]

28.15. Solve: (a) The current density is

\[ J = \frac{I}{A} = \frac{2.5 \text{ A}}{4.0 \times 10^{-5} \text{ m}^2} = 6.25 \times 10^5 \text{ A/m}^2 \]

(b) Using Equation 28.13 and Table 28.1, the drift speed is

\[ v_0 = \frac{J}{ne} = \frac{6.25 \times 10^5 \text{ A/m}^2}{(6.0 \times 10^{19} \text{ m}^{-3})(1.6 \times 10^{-19} \text{ C})} = 6.51 \times 10^{-3} \text{ m/s} \]

28.16. Visualize:

The current density \( J \) in a wire, as given by Equation 28.13, does not depend on the thickness of the wire.

Solve: (a) The current in the wire is

\[ I_{\text{wire}} = J_{\text{wire}} A_{\text{wire}} = (450,000 \text{ A/m}^2) \pi \left(\frac{1}{2}(1.5 \times 10^{-3} \text{ m})\right)^2 = 0.795 \text{ A} \]

Because current is continuous, \( I_{\text{wire}} = I_{\text{filament}} \). Thus, \( I_{\text{filament}} = 0.795 \text{ A} \).

(b) The current density in the filament is

\[ J_{\text{filament}} = \frac{I_{\text{filament}}}{A_{\text{filament}}} = \frac{0.795 \text{ A}}{\pi \left(\frac{1}{2}(0.12 \times 10^{-3} \text{ m})\right)^2} = 7.03 \times 10^7 \text{ A/m}^2 \]
28.17. **Solve:** (a) The current density is 
\[ J = \frac{I}{A} = \frac{0.85 \text{ A}}{\pi R^2} = \frac{0.85 \text{ A}}{\pi \left[ \frac{1}{2} (0.00025 \text{ m}) \right]^2} = 1.73 \times 10^7 \text{ A/m}^2 \]

(b) The electron current, or number of electrons per second, is 
\[ \frac{N_e}{\Delta t} = \frac{I}{e} = \frac{0.85 \text{ A}}{1.60 \times 10^{-19} \text{ C}} = \frac{0.85 \text{ C/s}}{1.60 \times 10^{-19} \text{ C}} = 5.31 \times 10^{16} \text{ s}^{-1} \]

28.18. **Solve:** (a) From Equation 28.13 and Table 28.1, the current density in the gold wire is 
\[ J = nev_d = (5.9 \times 10^{28} \text{ m}^{-3})(1.60 \times 10^{-19} \text{ C})(3.0 \times 10^{-4} \text{ m/s}) = 2.83 \times 10^6 \text{ A/m}^2 \]

(b) The current is 
\[ I = JA = (2.83 \times 10^6 \text{ A/m}^2)(0.25 \times \text{m}) = 0.556 \text{ A} \]

28.19. **Solve:** From Equation 28.13, the current in the wire is 
\[ I = JA = (7.50 \times 10^5 \text{ A/m}^2)(2.5 \times 10^{-6} \text{ m} \times 75 \times 10^{-6} \text{ m}) = 0.141 \text{ mA} \]

28.20. **Visualize:**

![Cross section of wire](image)

**Solve:** The current-carrying cross section of the wire is 
\[ A = \pi r_1^2 - \pi r_2^2 = \pi \left[ (0.0010 \text{ m})^2 - (0.0005 \text{ m})^2 \right] = 2.356 \times 10^{-6} \text{ m}^2 \]

The current density is 
\[ J = \frac{10 \text{ A}}{2.356 \times 10^{-6} \text{ m}^2} = 4.24 \times 10^6 \text{ A/m}^2 \]

28.21. **Model:** Use the model of conduction to relate the mean time between collisions to conductivity. 

**Solve:** From Equation 28.17, Table 28.1, and Table 28.2, the mean time between collisions for aluminum is 
\[ \tau_{\text{Al}} = \frac{m \sigma_{\text{Al}}}{n_{\text{Al}} e^2} = \frac{9.11 \times 10^{-31} \text{ kg}}{6.0 \times 10^{28} \text{ m}^{-3}} = 2.08 \times 10^{-14} \text{ s} \]

Similarly, the mean time between collisions for iron is 
\[ \tau_{\text{Fe}} = 4.19 \times 10^{-15} \text{ s} \]

**Assess:** The mean time between collisions in metals are of the order of $10^{-14}$ s.

28.22. **Model:** We will use the model of conduction to relate the mean time between collisions to conductivity. 

**Solve:** From Equation 28.17, Table 28.1, and Table 28.2, the mean time between collisions for silver is 
\[ \tau_{\text{Ag}} = \frac{m \sigma_{\text{Ag}}}{n_{\text{Ag}} e^2} = \frac{6.2 \times 10^{-31} \text{ kg}}{5.8 \times 10^{28} \text{ m}^{-3}} = 3.80 \times 10^{-14} \text{ s} \]

Similarly, the mean time between collisions for gold is 
\[ \tau_{\text{Au}} = 2.47 \times 10^{-14} \text{ s} \]

**Assess:** Mean free times in metals are of the order of $10^{-14}$ s.

28.23. **Solve:** The current density is 
\[ J = \sigma E \]

Using Equation 28.18 and Table 28.2, the current in the wire is 
\[ I = \sigma EA = (3.5 \times 10^{-1} \text{ M}^{-1})(0.012 \text{ N/C})(4 \times 10^{-6} \text{ m}^2) = 1.68 \text{ A} \]

28.24. **Solve:** From Equation 28.18 and Table 28.2, the electric field is 
\[ E = \frac{J}{\sigma} = \frac{1.0 \times 10^{-7} \text{ M}^{-1}}{5.0 \text{ A}} = 0.159 \text{ N/C} \]
28.25. Solve: From Equations 28.18 and 28.19, the resistivity is
\[
\rho = \frac{E}{J} = \frac{E}{I/A} = \frac{EA}{I} = \frac{(0.085 \text{ N/C}) \pi (1.5 \times 10^{-3} \text{ m})^2}{12 \text{ A}} = 5.01 \times 10^{-8} \Omega \text{ m}
\]

28.26. Solve: From Equations 28.18 and 28.19, the resistivity is
\[
\rho = \frac{E}{J} = \frac{E}{I/A} = \frac{EA}{I} = \frac{(0.0075 \text{ N/C}) \pi (0.5 \times 10^{-3} \text{ m})^2}{3.9 \times 10^{-3} \text{ A}} = 1.51 \times 10^{-6} \Omega \text{ m}
\]
From Table 8.2, we see that the wire is made of nichrome.

28.27. Solve: (a) Since \( J = \sigma E \) and \( J = I/A \), the electric field is
\[
E = \frac{I}{\sigma A} = \frac{I}{\pi \sigma} = \frac{0.020 \text{ A}}{\pi (0.25 \times 10^{-3} \text{ m})^2 (6.2 \times 10^7 \Omega^{-1} \text{ m}^{-1})} = 1.64 \times 10^{-3} \text{ N/C}
\]
(b) Since the current density is related to \( v_d \) by \( J = I/A = nev_d \), the drift speed is
\[
v_d = \frac{I}{\pi \sigma ne} = \frac{0.020 \text{ A}}{\pi (0.25 \times 10^{-3} \text{ m})^2 (5.8 \times 10^{28} \text{ m}^{-3})(1.60 \times 10^{-19} \text{ C})} = 1.10 \times 10^{-5} \text{ m/s}
\]
Assess: The values of \( n \) and \( \sigma \) for silver have been taken from Table 28.1 and Table 28.2. The drift velocity is typical of metals.

28.28. Visualize:
\[
\begin{align*}
\text{1 cm x 1 cm x 1 cm block} \quad E_{\text{vis}} \quad I \\
\text{1 cm x 1 cm x 1 cm block} \quad E_{\text{vis}} \quad I
\end{align*}
\]
Solve: The current density through the cube is \( J = \sigma E \), and the actual current is \( I = AJ \). Combining these equations, the conductivity is
\[
\sigma = \frac{I}{AE} = \frac{9.0 \text{ A}}{(10^{-3} \text{ m}^3)(5.0 \times 10^{-3} \text{ N/C})} = 1.8 \times 10^7 \text{ m}^{-1} \Omega^{-1}
\]
If all quantities entering the calculation are in SI units then the result for \( \sigma \) has to be in SI units. From the value for \( \sigma \), we can identify the metal as being tungsten.

28.29. Solve: The density of aluminum is 2700 kg/m³, so 1.0 m³ of aluminum has a mass of 2700 kg. The conduction-electron density is the number of electrons in 1.0 m³. Because the atomic mass of aluminum is 27 u, 27 g of aluminum contains \( N_A = 6.02 \times 10^{23} \) atoms. Thus, the number of aluminum atoms in 2700 kg of aluminum is
\[
\frac{6.02 \times 10^{23} \text{ (27 g)}}{(27 \text{ g})} = 6.0 \times 10^{28}
\]
The number density of aluminum atoms is \( 6.0 \times 10^{28} \text{ m}^{-3} \). Since each aluminum atom contributes one conduction electron to the metal, the number density of conduction electrons in aluminum is \( n = 6.0 \times 10^{28} \text{ m}^{-3} \).
Assess: The number density \( n \) obtained above agrees with that given in Table 28.1.

28.30. Solve: Equation 28.18 will be used to relate electric field strength with the diameter. We have
\[
J = \frac{I}{A} = \frac{I}{\frac{1}{4} \pi D^2} = \frac{4I}{\pi D^2} = \sigma E \Rightarrow I = \frac{\sigma \pi E D^2}{4}
\]
Because the current is the same in the two wires,
\[
E_{\text{Nihrome}} = \left( \frac{\sigma_{\text{aluminum}}}{\sigma_{\text{Nihrome}}} \cdot \frac{D_{\text{aluminum}}}{D_{\text{Nihrome}}} \right)^2 E_{\text{aluminum}}
\]
Using the values of $\sigma$ from Table 28.2,

$$E_{\text{actance}} = \left(3.5 \times 10^7 \frac{\Omega^{-1} \text{m}^{-1}}{6.7 \times 10^3 \frac{\Omega}{\text{m}^{-1}}} \right) \left(\frac{1.0 \text{ mm}}{2.0 \text{ mm}} \right)^2 \left(0.0080 \text{ N} / \text{C} \right) = 0.104 \text{ N/C}$$

### 28.31. Solve:

(a) Let the new current be $I'$ such that $I' = 2I$ where $I$ is the original current. The current density is $J = I/A$. Since the area of cross section of the wire remains the same, we have

$$A = \frac{I}{J} \Rightarrow J' = \left(\frac{I'}{I}\right)J = 2J$$

That is, doubling the current doubles the current density.

(b) Conduction-electron density is a material's characteristic value, and does not depend on external factors. Thus $n' = n$.

(c) The mean time between collisions is a material’s characteristic value, and does not depend on external factors. Thus, $\tau = \tau'$.

(d) From the model of conduction, Equation 28.7 is

$$v_a = \frac{e\tau}{m} E = \frac{e\tau}{m} \left(\frac{I}{\sigma A}\right) \Rightarrow v_a = \frac{v_a'}{I'} \Rightarrow v_a' = \frac{I'}{I}v_a = 2v_a$$

That is, doubling the current doubles the drift speed.

### 28.32. Solve:

(a) We will denote the new (or changed) quantities by primes. The new electric field in the wire is $E'$ such that $E' = 2E$. Because $J = I/A = \sigma E$, $I/E = \sigma A$ is a constant. Therefore

$$\frac{I}{E} = \frac{I'}{E'} \Rightarrow I' = \left(\frac{E'}{E}\right)I \Rightarrow I' = 2I$$

Thus, doubling the electric field doubles the current in the wire.

(b) The conduction-electron density is an intrinsic property of the material and cannot be changed by changing the external electric field.

(c) The mean time between collisions is an intrinsic property of the material and cannot be changed by changing the external electric field.

(d) From the model of conduction, Equation 28.7 is $v_a = \frac{e\tau E}{m}$. Because $e$, $\tau$, and $m$ do not change with electric field,

$$v_a' = \frac{e\tau}{m} E' = \frac{e\tau}{m} (2E) = 2v_a$$

That is, doubling the electric field doubles the drift speed.

### 28.33. Solve:

(a) The moving electrons are a current, even though they're not confined inside a wire. The electron current is

$$N_e = \frac{I}{\Delta t} = \frac{50 \times 10^{-6} \text{ A}}{1.60 \times 10^{-19} \text{ C}} = 3.12 \times 10^{14} \text{ s}^{-1}$$

This means during the time interval $\Delta t = 1 \text{ s}$, $3.12 \times 10^{14}$ electrons strike the screen.

(b) The current density is

$$J = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{50 \times 10^{-6} \text{ A}}{\pi(0.00020 \text{ m})^2} = 398 \text{ A} / \text{m}^2$$

(c) The acceleration can be found from kinematics:

$$v_f^2 = (4.0 \times 10^7 \text{ m/s})^2 = v_i^2 + 2a\Delta x \Rightarrow a = \frac{(4.0 \times 10^7 \text{ m/s})^2}{2(5.0 \times 10^{-7} \text{ m})} = 1.60 \times 10^{17} \text{ m/s}^2$$

But the acceleration is $a = F/m = eE/m$. Consequently, the electric field must be

$$E = \frac{ma}{e} = \frac{(9.11 \times 10^{-31} \text{ kg})(1.60 \times 10^{17} \text{ m/s}^2)}{1.6 \times 10^{-19} \text{ C}} = 9.11 \times 10^5 \text{ N/C}$$

(d) When they strike the screen, each electron has a kinetic energy

$$K = \frac{1}{2}mv_f^2 = \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(4.0 \times 10^7 \text{ m/s}) = 7.288 \times 10^{-16} \text{ J}$$
Power is the rate at which the screen absorbs this energy. The power of the beam is

\[ P = \frac{\Delta E}{\Delta t} = K \frac{N_e}{\Delta t} = \left(7.288 \times 10^{-16} \text{ J} \right) \left(3.12 \times 10^{14} \text{ s}^{-1} \right) = 0.227 \text{ J/s} = 0.227 \text{ W} \]

Assess: Power delivered to the screen by the electron beam is reasonable because the screen over time becomes a little warm.

28.34. Visualize: Please refer to Figure P28.34.

Solve: (a) The current associated with the moving film is the rate at which the charge on the film moves past a certain point. The tangential speed of the film is

\[ v = \omega r = (90 \text{ rpm}) \left(4.0 \text{ cm} \right) = 90 \text{ rev/min} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times 1.0 \text{ cm} = 9.425 \text{ cm/s} \]

In 1.0 s the film moves a distance of 9.425 cm. This means the area of the film that moves to the right in 1.0 s is 
\[ (9.425 \text{ cm})(4.0 \text{ cm}) = 37.7 \text{ cm}^2 \]

The amount of charge that passes to the right in 1.0 s is

\[ Q = (37.7 \text{ cm}^2)(-2.0 \times 10^{-9} \text{ C/cm}^2) = -75.4 \times 10^{-9} \text{ C} \]

Since \( I = Q/\Delta t \), we have

\[ I = \frac{-75.4 \times 10^{-9} \text{ C}}{1 \text{ s}} = 75.4 \text{ nA} \]

(b) Having found the current in part (a), we can once again use \( I = Q/\Delta t \) to obtain \( \Delta t \):

\[ \Delta t = \frac{Q}{I} = \frac{-75.4 \times 10^{-9} \text{ C}}{75.4 \times 10^{-9} \text{ A}} = 133 \text{ s} \]

28.35. Model: The current is the rate at which the charge of the ions moves through the ionic solution.

Solve: Because the atomic mass of gold is 197 g, the number of gold atoms is

\[ N = \frac{M}{M_A} N_A = \left(\frac{0.5 \text{ g}}{197 \text{ g mol}^{-1}}\right) 6.02 \times 10^{23} \text{ mol}^{-1} = 1.528 \times 10^{21} \text{ atoms} \]

We need to deposit \( N = 1.528 \times 10^{21} \) gold ions, each with a charge of \(-1.60 \times 10^{-19} \text{ C}\), in 3 hours on the statue. The current is

\[ I = \frac{Q}{\Delta t} = \frac{(1.528 \times 10^{21})(1.60 \times 10^{-19} \text{ C})}{3 \times 3600 \text{ s}} = 22.6 \text{ mA} \]

28.36. Solve: (a) A current of 1.8 pA for the potassium ions means that a charge of 1.8 pC flows through the potassium ion channel per second. The number of potassium ions that pass through the ion channel per second is

\[ \frac{1.8 \times 10^{-12} \text{ C/s}}{1.6 \times 10^{-19} \text{ C}} = 1.125 \times 10^7 \text{ s}^{-1} \]

Since the channel opens only for 1.0 ms, the total number of potassium ions that pass through the channel is

\[ (1.125 \times 10^7 \text{ s}^{-1})(1.0 \times 10^{-3} \text{ s}) = 1.13 \times 10^5 \text{ atoms} \]

(b) The current density in the ion channel is

\[ J = \frac{I}{A} = \frac{1.8 \text{ pA}}{\pi(0.30 \text{ nm/2})^2} = \frac{1.8 \times 10^{-12} \text{ A}}{\pi(0.15 \times 10^{-8} \text{ m})^2} = 2.55 \times 10^7 \text{ A/m}^2 \]

28.37. Solve: (a) Current is defined as \( I = Q/\Delta t \), so the charge delivered in time \( \Delta t \) is

\[ Q = I \Delta t = (150 \text{ A})(0.80 \text{ s}) = 120 \text{ C} \]

(b) The drift speed is

\[ v = \frac{J}{ne} = \frac{I}{ne} = \frac{150 \text{ A}}{\pi(0.0025 \text{ m})^2(8.5 \times 10^{26} \text{ m}^{-1})(1.60 \times 10^{-19} \text{ C})} = 5.617 \times 10^{-4} \text{ m/s} \]

At this speed the electrons drift a distance

\[ d = (5.617 \times 10^{-4} \text{ m/s})(0.80 \text{ s}) = 4.49 \times 10^{-4} \text{ m} = 0.449 \text{ mm} \]
28.38. Solve: The total charge in the battery is
\[ Q = I \Delta t = (90 \text{ A})(3600 \text{ s}) = 3.2 \times 10^5 \text{ C} \]

28.39. Model: We assume that the charge carriers are uniformly distributed throughout the wire.
Solve: Using Equation 28.16, we can write the current density as
\[ J = \frac{I}{A} = \frac{ne^2 \tau}{m} E \Rightarrow I = \left( \frac{ne^2 \tau}{m} E \right) A \]
For a given wire, the current is thus proportional to the area through which the current flows: \( I_{\text{total}} \propto R^2 \) and \( I_{\text{center}} \propto (\frac{1}{4}R)^2 \). Therefore,
\[ \frac{I_{\text{center}}}{I_{\text{total}}} = \frac{1}{4} = 25\% \]
That is, 25% of the total current flows in the part of the wire with radius \( r \leq R/2 \).

28.40. Solve: Equation 28.13 defines the current density as \( J = I/A \). This means
\[ A = \frac{\pi D^2}{4} = \frac{I}{J} \Rightarrow D = \sqrt{\frac{4I}{\pi J}} = \sqrt{\frac{4(1.0 \text{ A})}{\pi(500 \text{ A/cm}^2)}} = 0.050 \text{ cm} = 0.50 \text{ mm} \]
Assess: Fuse wires are usually thin.

28.41. Solve: The density of copper \( \rho = 8940 \text{ kg/m}^3 = 8.94 \text{ g/cm}^3 \). The volume of the hollow copper cylinder is
\[ V = \frac{M}{\rho} = \frac{62.0 \text{ g}}{8.94 \text{ g/cm}^3} = 6.935 \text{ cm}^3 \]
Because cylinder’s length is 10 cm, the area of cross section of the hollow cylinder is
\[ A = \frac{V}{L} = \frac{6.935 \text{ cm}^3}{10 \text{ cm}} = 0.6935 \text{ cm}^2 \]
Thus, the current \( I = JA = (150,000 \text{ A/m}^2)(0.6935 \times 10^{-4} \text{ m}^2) = 10.4 \text{ A} \).

28.42. Visualize:

Solve: (a) Consider a cylindrical surface inside the metal at a radial distance \( r \) from the center. The current is flowing through the walls of this cylinder, which have surface area \( A = (2\pi r)L \). Thus
\[ I = JA = \sigma E (2\pi r) L \]
Thus the electric field strength at radius \( r \) is
\[ E = \frac{I}{2\pi \sigma r L} \]
(b) For iron, with $\sigma = 1.0 \times 10^7 \, \Omega^{-1} \text{m}^{-1}$,

$$E_{outer} = \left( \frac{25 \text{ A}}{2\pi (0.10 \text{ m})(1.0 \times 10^7 \, \Omega^{-1} \text{m}^{-1})} \right) \left( \frac{1}{0.01 \text{ m}} \right) = 3.98 \times 10^{-5} \, \text{N/C}$$

$$E_{outer} = \left( \frac{25 \text{ A}}{2\pi (0.25 \text{ m})(1.0 \times 10^7 \, \Omega^{-1} \text{m}^{-1})} \right) \left( \frac{1}{0.025 \text{ m}} \right) = 1.59 \times 10^{-5} \, \text{N/C}$$

28.43. Visualize:

Solve: (a) Consider a spherical surface inside the hollow sphere at a radial distance $r$ from the center. The current is flowing outward through this surface, which has surface area $A = 4\pi r^2$. Thus

$$I = JA = \sigma E(4\pi r^2)$$

Thus the electric field strength at radius $r$ is

$$E = \frac{I}{4\pi \sigma r^2}$$

(b) For copper, with $\sigma = 6.0 \times 10^7 \, \Omega^{-1} \text{m}^{-1}$.

$$E_{outer} = \frac{25 \text{ A}}{4\pi (6 \times 10^7 \, \Omega^{-1} \text{m}^{-1}) (0.01 \text{ m})^2} = 3.32 \times 10^{-4} \, \text{N/C}$$

$$E_{outer} = \frac{25 \text{ A}}{4\pi (6 \times 10^7 \, \Omega^{-1} \text{m}^{-1}) (0.025 \text{ m})^2} = 5.31 \times 10^{-5} \, \text{N/C}$$

28.44. Solve: (a)

(b) Since $I = \Delta Q/\Delta t$, for infinitesimal changes

$$I = \frac{dQ}{dt} = \frac{d}{dt} (4t - t^2) = 4 - 2t$$

(c)

(d) The value of $I$ at $t = 2.0$ s is zero. This is because the charge, having reached its maximum value, has stopped entering the wire. The negative values of $I$ mean that charge is flowing out of the wire.
28.45. Solve: (a) The value of $Q$ (C) for selected values of $t$ are

<table>
<thead>
<tr>
<th>$t$ (s)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$ (C)</td>
<td>7.87</td>
<td>12.6</td>
<td>17.3</td>
<td>19.0</td>
<td>19.6</td>
<td>19.9</td>
<td></td>
</tr>
</tbody>
</table>

(b) Since $I = \Delta Q/\Delta t$, for infinitesimal changes

$$I = \frac{dQ}{dt} = \frac{d}{dt} (20 \text{ C})(1 - e^{-t/2.0 \text{s}}) = (20 \text{ C})(-e^{-t/2.0 \text{s}})(-1/2.0 \text{s}) = (10 \text{ A})e^{-t/2.0 \text{s}}$$

(c) The maximum value of the current occurs at $t = 0$ s and is 10.0 A.

(d) The values of $I$ (A) for selected values of $t$ are

<table>
<thead>
<tr>
<th>$t$ (s)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$ (A)</td>
<td>10</td>
<td>6.07</td>
<td>3.68</td>
<td>1.35</td>
<td>0.50</td>
<td>0.18</td>
<td>0.07</td>
</tr>
</tbody>
</table>

28.46. Solve: (a) The values of $I$ (A) for selected values of $t$ are

<table>
<thead>
<tr>
<th>$t$ (µs)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$ (A)</td>
<td>2.0</td>
<td>1.21</td>
<td>0.74</td>
<td>0.27</td>
<td>0.10</td>
<td>0.036</td>
<td>0.014</td>
</tr>
</tbody>
</table>

(b) Because $I = dQ/dt$,

$$Q = \int I dt = \int_0^t (2.0 \text{ A})e^{-t/(2.0 \mu \text{s})} dt = \left[-4.0 \mu \text{C}e^{-t/(2.0 \mu \text{s})}\right]_0 = (4.0 \mu \text{C})[1 - e^{-t/(2.0 \mu \text{s})}]$$

where we have used the condition $Q = 0$ C at $t = 0$ µs.

(c) The values of $Q$ (µC) for selected values of $t$ are

<table>
<thead>
<tr>
<th>$t$ (µs)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$ (µC)</td>
<td>0</td>
<td>1.57</td>
<td>2.52</td>
<td>3.46</td>
<td>3.80</td>
<td>3.92</td>
<td>3.98</td>
</tr>
</tbody>
</table>
Both the rings contribute equally to the field strength. The radius of each ring is \( R = 1.5 \) mm. The left ring is negatively charged and the right ring is positively charged because 20 electrons have been transferred from the right ring to the left ring.

**Solve:** From Chapter 26, the electric field of a ring of charge \( +Q \) at \( z = -1.0 \) mm on the axis where the axis of each ring is \( k \) is

\[
\mathbf{E}_z = \frac{1}{4\pi\varepsilon_0} \frac{z Q}{(z^2 + R^2)^{3/2}} \mathbf{\hat{k}} = \left(9 \times 10^8 \text{ N m}^2/\text{C}^2\right) \left(\frac{(-1.0 \times 10^{-3} \text{ m})(20 \times 1.6 \times 10^{-19} \text{ C})}{(-1.0 \times 10^{-3} \text{ m})^2 + (1.5 \times 10^{-3} \text{ m})^2}\right)^{3/2} \mathbf{\hat{k}}
\]

\[
= -4.92 \times 10^{-3} \mathbf{\hat{k}} \text{ N/C}
\]

The left ring with charge \(-Q\) makes an equal contribution

\[
\mathbf{E}_z = -4.92 \times 10^{-3} \mathbf{\hat{k}} \text{ N/C}
\]

\[
\Rightarrow \mathbf{E}_\text{net} = \mathbf{E}_+ + \mathbf{E}_- = -9.84 \times 10^{-3} \mathbf{\hat{k}} \text{ N/C}
\]

The negative sign with \( E_+ \), \( E_- \), and \( E_\text{net} \) means these electric fields are in the \(-z\) direction. Using \( J = \sigma E \), the current is

\[
I = \pi (1.5 \times 10^{-3} \text{ m})^2 (3.5 \times 10^7 \text{ \Omega}^{-1} \text{m}^{-1})(9.84 \times 10^{-3} \text{ N/C}) = 2.43 \text{ A}
\]

**Assess:** This result is consistent with the value given in Table 26.1 for the electric field strength in a current-carrying wire.

28.48. **Model:** Because current is conserved, the current flowing in the 2.0-mm-diameter segment of the wire is the same as in the 1.0-mm-diameter segment.

**Visualize:** Please refer to Figure P28.48. We will denote all quantities for the 1.0-mm-diameter wire with the subscript 1, and all quantities for the 2.0-mm-diameter wire with the subscript 2.

**Solve:** Equation 28.13 is \( J = nev_\text{e} \). This means the current densities in the two segments are

\[
J_1 = nev_{\text{e}1} \quad J_2 = nev_{\text{e}2}
\]

Dividing these equations, we get \( v_{\text{e}1} = (J_2/J_1)v_{\text{e}1} \). Because current is conserved, \( I_1 = I_2 = 2.0 \text{ A} \). So,

\[
\frac{J_2}{J_1} = \frac{I_2/A_1}{I_1/A_1} = \frac{A_1}{A_2} \Rightarrow v_{\text{e}1} = \frac{A_1}{A_2} v_{\text{e}1} = \left(\frac{D_1}{D_2}\right)^2 \left(\frac{1.0 \text{ mm}}{2.0 \text{ mm}}\right)^2 (2.0 \times 10^{-4} \text{ m/s}) = 5.0 \times 10^{-4} \text{ m/s}
\]

**Assess:** A drift velocity which is small and only \((\frac{1}{2})\) of the drift velocity in the 1.0-mm-diameter wire is reasonable.

28.49. **Model:** Because current is conserved, the current in the 3.0-mm-diameter end of the wire is the same as in the current in the 1.0-mm-diameter end of the wire.

**Visualize:** Please refer to Figure P28.49. We will denote all quantities for the 1.0-mm-diameter end of the wire with the subscript 1, and all quantities for the 3.0-mm-diameter end of the wire with the subscript 3.

**Solve:** Equation 28.13 is \( J = nev_\text{e} \). This means the current densities at the two ends are

\[
J_1 = nev_{\text{e}1} \quad J_3 = nev_{\text{e}3}
\]

Dividing these equations, we obtain \( v_{\text{e}1} = (J_3/J_1)v_{\text{e}1} \). Because current is conserved, \( I_1 = I_3 \). So,

\[
v_{\text{e}1} = \frac{I_3/A_1}{I_1/A_3} v_{\text{e}1} = \frac{A_1}{A_3} v_{\text{e}1} = \left(\frac{D_1}{D_3}\right)^2 \left(\frac{1.0 \text{ mm}}{3.0 \text{ mm}}\right)^2 (0.5 \times 10^{-4} \text{ m/s}) = 5.6 \times 10^{-6} \text{ m/s}
\]

**Assess:** The smallness of the drift velocity is physically reasonable.
28.50. **Model:** Because current is conserved, the currents in the aluminum and the nichrome segments of the wire are the same.

**Visualize:** Please refer to Figure P28.50.

**Solve:** From Equations 28.13 and 28.18 we have \( J = I/A = \sigma E \). This means

\[
E = \frac{J}{A\sigma} = \frac{L_{\text{nichrome}}}{E_{\text{aluminum}}} = \left( \frac{L_{\text{nichrome}}}{L_{\text{aluminum}}} \right) \left( \frac{\sigma_{\text{aluminum}}}{\sigma_{\text{nichrome}}} \right) = \left( \frac{D_{\text{nichrome}}^2}{D_{\text{aluminum}}^2} \right) \left( \frac{\sigma_{\text{aluminum}}}{\sigma_{\text{nichrome}}} \right)
\]

where we used the conservation of current and \( A = \frac{\pi}{4} D^2 \). For \( E_{\text{nichrome}} = E_{\text{aluminum}} \), the above equation simplifies to

\[
D_{\text{nichrome}} = \sqrt{\frac{\sigma_{\text{aluminum}}}{\sigma_{\text{nichrome}}} D_{\text{aluminum}}} = \sqrt{\frac{3.5 \times 10^{-7} \text{ m}^{-1}}{6.7 \times 10^{-7} \text{ m}^{-1}} (1.0 \text{ mm})} = 7.22 \text{ mm}
\]

28.51. **Model:** Because current is conserved, the currents in the two segments of the wire are the same.

**Visualize:** Please refer to Figure P28.51.

**Solve:** The currents in the two segments of the wire are related by \( I_1 = I_2 \). But \( I = AJ \), so we have \( A_1 J_1 = A_2 J_2 \).

The wire’s diameter is constant, so \( J_1 = J_2 \) and \( \sigma_1 E_1 = \sigma_2 E_2 \). The ratio of the electric fields is

\[
\frac{E_1}{E_2} = \frac{\sigma_1}{\sigma_2} = \frac{1}{2}
\]

28.52. **Model:** Electric current is conserved.

**Visualize:** Please refer to Figure P28.52. For the top, middle, and bottom segments the subscripts “top,” “mid,” and “bot” are used.

**Solve:** (a) Since current is conserved, \( I_{\text{top}} = I_{\text{bot}} = I_{\text{mid}} = 10 \text{ A} \).

(b) The current density is \( J_{\text{top}} = \frac{10 \text{ A}}{\pi(0.001 \text{ m})^2} = 3.18 \times 10^6 \text{ A/m}^2 \) \( J_{\text{bot}} = \frac{10 \text{ A}}{\pi(0.0005 \text{ m})^2} = 1.27 \times 10^7 \text{ A/m}^2 \)

(c) The electric field is \( E = J/\sigma \). Thus,

\[
E_{\text{top}} = \frac{J_{\text{top}}}{\sigma} = \frac{3.18 \times 10^6 \text{ A/m}^2}{3.5 \times 10^{-7} \text{ m}^{-1}} = 0.0909 \text{ N/C} \quad E_{\text{bot}} = \frac{J_{\text{bot}}}{\sigma} = \frac{1.27 \times 10^7 \text{ A/m}^2}{3.5 \times 10^{-7} \text{ m}^{-1}} = 0.364 \text{ N/C}
\]

(d) The drift speed is \( v_d = J/ne \). Thus

\[
(v_d)_{\text{top}} = \frac{(v_d)_{\text{bot}}}{n_e} = \frac{J_{\text{top}}}{n_e} = \frac{3.18 \times 10^6 \text{ A/m}^2}{(6.0 \times 10^{25} \text{ m}^{-3})(1.60 \times 10^{-19} \text{ C})} = 3.31 \times 10^{-4} \text{ m/s}
\]

\[
(v_d)_{\text{mid}} = \frac{(v_d)_{\text{bot}}}{n_e} = \frac{J_{\text{bot}}}{n_e} = \frac{1.27 \times 10^7 \text{ A/m}^2}{(6.0 \times 10^{25} \text{ m}^{-3})(1.60 \times 10^{-19} \text{ C})} = 1.33 \times 10^{-3} \text{ m/s}
\]

(e) The mean time between collisions is \( \tau = mv/eE \). Thus,

\[
\tau_{\text{top}} = \frac{m(v_d)_{\text{top}}}{eE_{\text{top}}} = \frac{(9.11 \times 10^{-31} \text{ kg})(3.31 \times 10^{-4} \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.0909 \text{ N/C})} = 2.07 \times 10^{-14} \text{ s}
\]

\[
\tau_{\text{bot}} = \frac{m(v_d)_{\text{bot}}}{eE_{\text{bot}}} = \frac{(9.11 \times 10^{-31} \text{ kg})(1.33 \times 10^{-3} \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.364 \text{ N/C})} = 2.08 \times 10^{-14} \text{ s}
\]

The collision times are the same in all three segments because collisions are part of the microphysics of electrons inside the metal, independent of the macrophysics of fields and currents.

(f) The electron current is \( I = N_e/\Delta t = I/e \). Because \( I_1 = I_2 = I_3 = 10 \text{ A} \),

\[
\left( \frac{N_e}{\Delta t} \right)_{\text{top}} = \left( \frac{N_e}{\Delta t} \right)_{\text{bot}} = \left( \frac{N_e}{\Delta t} \right)_{\text{mid}} = \frac{10 \text{ A}}{1.60 \times 10^{-19} \text{ C}} = 6.25 \times 10^{10} \text{ s}^{-1}
\]
28.53. **Solve:** The total charge delivered by the battery is

\[
Q = \int_0^6 I(t) \, dt = \int_0^6 (0.75 \, \text{A}) e^{-\frac{t}{6 \, \text{hours}}} \, dt = (0.75 \, \text{A})(-6 \, \text{hours})\left[e^{-\frac{6 \, \text{hours}}{6 \, \text{hours}}}ight]_0 \\
= (0.75 \, \text{A})(6 \times 3600 \, \text{s}) = 1.62 \times 10^4 \, \text{C}
\]

The number of electrons transported is

\[
\frac{1.62 \times 10^4 \, \text{C}}{1.6 \times 10^{19} \, \text{C}} = 1.01 \times 10^{15}
\]

28.54. **Model:** Current is the rate at which the charge moves across a certain cross section.

**Solve:** (a) For the circular motion of the electron around the proton, Coulomb's force between the electron and the proton causes the centripetal acceleration. Thus,

\[
\frac{1}{4\pi e_0} \frac{|-e| v^2}{r} = \frac{m}{r} \left(\frac{2\pi}{T}\right)^2 \Rightarrow \frac{1}{T} = f = \frac{\left(9.0 \times 10^9 \, \text{N m}^2 / \text{C}^2\right)\left(1.6 \times 10^{-19} \, \text{C}\right)^2}{4\pi^2 \left(0.053 \times 10^{-9} \, \text{m}\right)\left(9.11 \times 10^{-31} \, \text{kg}\right)} = 6.56 \times 10^{15} \, \text{Hz}
\]

(b) Charge \(Q = e\) passes any point on the orbit once every period. Thus the effective current is

\[
I = \frac{Q}{\Delta T} = \frac{e}{T} = ef = \left(1.6 \times 10^{-19} \, \text{C}\right)\left(6.56 \times 10^{15} \, \text{Hz}\right) = 1.05 \times 10^{-3} \, \text{A}
\]

28.55. **Model:** Assume that the conduction electrons are point particles.

**Solve:** (a) The rms velocity of the conduction electrons at room temperature is obtained from the relationship \(\frac{1}{2} mv_{rms}^2 = \frac{1}{2} k_B T\). We have

\[
v_{rms} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3\left(1.38 \times 10^{-23} \, \text{J/K}\right)(293 \, \text{K})}{9.11 \times 10^{-31} \, \text{kg}}} = 1.15 \times 10^5 \, \text{m/s}
\]

(b) The mean free path of an electron depends on the number of atoms of copper per volume. The electron will collide with an atom if it comes within a distance \(r\) from the atom, where \(r\) is the radius of a copper atom. From Chapter 18, the mean free path is

\[
\lambda = \frac{L}{N_{\text{collisions}}} = \frac{L}{(N/V)\pi r^2} = \frac{1}{(N/V)\pi r^2}
\]

For atoms, \(r = 0.5 \times 10^{-10} \, \text{m}\). Using \(N/V = 8.5 \times 10^{28} \, \text{m}^{-3}\) from Table 28.1,

\[
\lambda = \frac{1}{(8.5 \times 10^{28})\pi(0.5 \times 10^{-10})^2} = 1.5 \times 10^{-9} \, \text{m} = 1.5 \, \text{nm}
\]

28.56. **Visualize:**

The current density in the beam increases with distance from the center. We consider a thin circular shell of width \(dr\) at a distance \(r\) from the center to calculate the current density at the edge.
Solve: (a) The beam current is 1.5 mA. This means the beam transports a charge of $1.5 \times 10^{-3} \text{ C}$ in 1 s. The number of protons delivered in one second is

$$\frac{1.5 \times 10^{-3} \text{ C}}{1.6 \times 10^{-19} \text{ C}} = 9.375 \times 10^{15}$$

(b) From $J = I/A$, the current in the ring of width $dr$ at a distance $r$ from the center is

$$dl = JdA = J_{\text{edge}} \left( \frac{r}{R} \right) (2\pi r dr) = J_{\text{edge}} \frac{2\pi r^2 dr}{R}$$

The total current $I = 1.5 \text{ mA}$ is found by integrating this expression:

$$I_{\text{tot}} = \int dl = 1.5 \times 10^{-3} \frac{A}{R} \int_0^R 2\pi r^2 dr = \frac{2\pi J_{\text{edge}} R^2}{3}$$

$$\Rightarrow J_{\text{edge}} = \left( 1.5 \times 10^{-3} \frac{\text{A}}{\text{m}} \right) \frac{3}{2\pi (2.5 \times 10^{-3} \text{ m})^2} = 115 \text{ A/m}^2$$

28.57. Model: The currents in the two segments of the wire are the same.

Visualize: The electric fields $E_1$ and $E_2$ point in the direction of the current. Establish a cylindrical Gaussian surface with end area $a$ that extends into both segments of the wire.

Solve: (a) Because current is conserved, $I_1 = I_2 = I$. The cross-section areas of the two wires are the same, so the current densities are the same: $J_1 = J_2 = I/A$. Thus the electric fields in the two segments have strengths

$$E_1 = J_1 = I = \frac{1}{A} \frac{1}{\sigma_1} \quad E_2 = J_2 = I = \frac{1}{A} \frac{1}{\sigma_2}$$

The electric field enters the Gaussian surface on the left (negative flux) and exits on the right. No flux passes through the wall of cylinder, so the net flux is $\Phi_e = E_1 a - E_1 a$. The Gaussian cylinder encloses charge $Q_m = \eta a$ on the boundary between the segments. Gauss’s law is

$$\Phi_e = \frac{Q_m}{\varepsilon_0} \Rightarrow E_2 a - E_1 a = \frac{I a}{A} \left( \frac{1}{\sigma_2} - \frac{1}{\sigma_1} \right) = \frac{\eta a}{\varepsilon_0}$$

Thus the surface charge density on the boundary is

$$\eta = \frac{\varepsilon_0 I}{A} \left( \frac{1}{\sigma_2} - \frac{1}{\sigma_1} \right)$$

(b) From the expression obtained in part (a)

$$\eta = \frac{Q}{\pi R^2} = \frac{I e_0}{(2\pi)^2} \left( \frac{1}{\sigma_{\text{iron}}} - \frac{1}{\sigma_{\text{copper}}} \right)$$

$$\Rightarrow Q = (5 \text{ A}) \left( 8.85 \times 10^{-12} \text{ C}^2 / \text{Nm}^2 \right) \left( \frac{1}{1.0 \times 10^{-7} \text{ S}^{-1} \text{m}^{-1}} - \frac{1}{6.0 \times 10^{-7} \text{ S}^{-1} \text{m}^{-1}} \right) = 3.68 \times 10^{-18} \text{ C}$$

Assess: This charge corresponds to a deficit of a mere 23 electrons on the boundary between the metals.