29.1. **Model:** The mechanical energy of the proton is conserved. A parallel plate capacitor has a uniform electric field.

**Visualize:**

- After: 
  - $v_f = 0 \text{ m/s}$
  - $x_f$
- Before: 
  - $v_i = v_i$
  - $x_i$

The figure shows the before-and-after pictorial representation. The proton has an initial speed $v_i = 0 \text{ m/s}$ and a final speed $v_f$ after traveling a distance $d = 2.0 \text{ mm}$.

**Solve:** The proton loses potential energy and gains kinetic energy as it moves toward the negative plate. The potential energy $U$ is defined as $U = U_0 + qEx$, where $x$ is the distance from the negative plate and $U_0$ is the potential energy at the negative plate (at $x = 0 \text{ m}$). Thus, the change in the potential energy of the proton is

$$
\Delta U = U_f - U_i = (U_0 + 0) - (U_0 + qEd) = -qEd
$$

The change in the kinetic energy of the proton is

$$
\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2
$$

The law of conservation of energy is $\Delta K + \Delta U = 0 \text{ J}$. This means

$$
\frac{1}{2}mv_f^2 + (-qEd) = 0 \text{ J}
$$

$$
\Rightarrow v_f = \sqrt{\frac{2qEd}{m}} = \sqrt{\frac{2(+1.60 \times 10^{-19} \text{ C})(50,000 \text{ N/C})(2.0 \times 10^{-3} \text{ m})}{(1.67 \times 10^{-27} \text{ kg})}} = 1.38 \times 10^3 \text{ m/s}
$$

**Assess:** As described in Section 29.1, the potential energy for a charge $q$ in an electric field $E$ is $U = U_0 + qEx$, where $x$ is the distance measured from the negative plate. Having $U = U_0$ at the negative plate (with $x = 0 \text{ m}$) is completely arbitrary. We could have taken it to be zero. Note that only $\Delta U$, and not $U$, has physical consequences.

29.2. **Model:** The mechanical energy of the electron is conserved. A parallel plate capacitor has a uniform electric field.

**Visualize:**

- Before: 
  - $v_i = 0 \text{ m/s}$
  - $x_i$
- After: 
  - $v_f$
  - $x_f$

- $x (\text{ mm})$

0 0.5 1.0
Chapter 29

The figure shows the before-and-after pictorial representation. The electron has an initial speed \( v_i = 0 \) m/s and a final speed \( v_f \) after traveling a distance \( d = 1.0 \) mm.

**Solve:** The electron loses potential energy and gains kinetic energy as it moves toward the positive plate. The potential energy \( U \) is defined as \( U = U_0 + qEx \), where \( x \) is the distance from the negative plate and \( U_0 \) is the potential energy at the negative plate (at \( x = 0 \) m). Thus, the change in the potential energy of the electron is

\[
\Delta U_e = U_f - U_i = (U_0 + qEd) - (U_0 + 0 \text{ J}) = qEd
\]

The change in the kinetic energy of the electron is

\[
\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2
\]

Now, the law of conservation of mechanical energy gives \( \Delta K + \Delta U = 0 \text{ J} \). This means

\[
\frac{1}{2}mv_f^2 + qEd = 0 \text{ J}
\]

\[
\Rightarrow v_f = \sqrt{\frac{-2qEd}{m}} = \sqrt{\frac{(-2)(-1.60 \times 10^{-19} \text{ C})(20,000 \text{ N/C})(1.0 \times 10^{-3} \text{ m})}{9.11 \times 10^{-31} \text{ kg}}} = 2.65 \times 10^6 \text{ m/s}
\]

**Assess:** Note that \( \Delta U_e = qEd \) is the change in the potential energy of the electron. It is negative because \( q = -e \) for the electron. Thus, the potential energy becomes more negative as \( d \) increases, that is, the potential energy of the electron decreases with an increase in \( d \) (or \( x \)).

29.3. **Model:** The mechanical energy of the proton is conserved. A parallel plate capacitor has a uniform electric field.

**Visualize:**

<table>
<thead>
<tr>
<th>Charge on each plate is ( Q )</th>
<th>Charge on each plate is ( 2Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Before" /></td>
<td><img src="image2" alt="Before" /></td>
</tr>
<tr>
<td><img src="image3" alt="After" /></td>
<td><img src="image4" alt="After" /></td>
</tr>
</tbody>
</table>

The figure shows the before-and-after pictorial representation.

**Solve:** The proton loses potential energy and gains kinetic energy as it moves toward the negative plate. The potential energy is defined as \( U = U_0 + qEx \), where \( x \) is the distance from the negative plate and \( U_0 \) is the potential energy at the negative plate (at \( x = 0 \) m). Thus, the change in the potential energy of the proton is

\[
\Delta U_p = U_f - U_i = (U_0 + 0 \text{ J}) - (U_0 + qEd) = -qEd
\]

The change in the kinetic energy of the proton is

\[
\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2
\]

Applying the law of conservation of energy \( \Delta K + \Delta U = 0 \text{ J} \), we have

\[
\frac{1}{2}mv_f^2 + (-qEd) = 0 \Rightarrow v_f^2 = \frac{2qEd}{m}
\]

When the amount of charge on each plate is doubled, then the final velocity of the proton is

\[
v_f' = \frac{2q'E'd}{m}
\]

Dividing these equations,

\[
\frac{v_f'^2}{v_f^2} = \frac{E'}{E} \Rightarrow v_f' = \sqrt{\frac{E'}{E}} v_f
\]

For a parallel-plate capacitor \( E = \eta/e_0 = Q/Ae_0 \). Therefore,

\[
v_f' = \sqrt{\frac{Q'}{Q}} v_f = \sqrt{2}(50,000 \text{ m/s}) = 7.07 \times 10^4 \text{ m/s}
\]

**Assess:** The proton’s velocity is expected to increase because an increased charge on the capacitor plates leads to a higher electric field between the plates and hence to an increased force on the proton.
29.4. Model: The mechanical energy of the charged particles is conserved. A parallel plate capacitor has a uniform electric field.

Visualize:

![Proton and Helium ion diagrams](image)

The figure shows the before-and-after pictorial representation.

Solve: The potential energy is defined as $U = U_0 + qE_d$, where $x$ is the distance from the negative plate and $U_0$ is the potential energy at the negative plate (at $x = 0$ m). Thus, the change in the potential energy of the proton as it moves from the positive plate to the negative plate is

$$\Delta U_p = U_f - U_i = (U_0 + eEd) - (U_0 + eEd) = -eEd$$

This decrease in potential energy appears as an increase in the proton's kinetic energy:

$$\Delta K = K_f - K_i = \frac{1}{2} m_p v_{ip}^2 - \frac{1}{2} m_p v_{it}^2$$

Applying the law of conservation of mechanical energy $\Delta K + \Delta U_p = 0$, we have

$$\frac{1}{2} m_p v_{ip}^2 + (-eEd) = 0 \Rightarrow v_{ip}^2 = \frac{2eEd}{m_p}$$

When the proton is replaced with a helium ion and the same experiment is repeated,

$$v_{i\text{ion}}^2 = \frac{2eEd}{m_{\text{ion}}}$$

Dividing the two equations,

$$v_{i\text{ion}} = \sqrt{\frac{m_p}{m_{\text{ion}}}} v_{ip} = \sqrt{\frac{1}{3}} (50,000 \text{ m/s}) = 2.5 \times 10^4 \text{ m/s}$$

Assess: Being a heavier particle, the helium ion's velocity is expected to be smaller compared to the proton's velocity.

29.5. Model: The charges are point charges.

Visualize: Please refer to Figure Ex29.5.

Solve: For a system of point charges, the potential energy is the sum of the potential energies due to all distinct pairs of charges:

$$U_{\text{ele}} = \sum_{ij} \frac{K q_i q_j}{r_{ij}} = U_{12} + U_{13} + U_{23}$$

$$= (9 \times 10^9 \text{ N m}^2 / \text{C}^2)(2 \times 10^{-2} \text{ C})(2 \times 10^{-4} \text{ C}) \left[ \frac{1}{0.03 \text{ m}} + \frac{1}{0.04 \text{ m}} + \frac{1}{\sqrt{(0.03 \text{ m})^2 + (0.04 \text{ m})^2}} \right]$$

$$= 1.29 \times 10^{-6} \text{ J} + 0.90 \times 10^{-6} \text{ J} + 0.72 \times 10^{-6} \text{ J} = 2.82 \times 10^{-6} \text{ J}$$

Assess: Note that $U_{12} = U_{23}$, $U_{13} = U_{31}$, and $U_{23} = U_{32}$.

29.6. Model: The charges are point charges.

Visualize: Please refer to Figure Ex29.6.
Solve: For a system of point charges, the potential energy is the sum of the potential energies due to all pairs of charges:

\[
U_{\text{elec}} = \sum_{i<j} \frac{q_i q_j}{r_{ij}} = U_{12} + U_{13} + U_{23} = K \frac{q_1 q_2}{r_{12}} + K \frac{q_1 q_3}{r_{13}} + K \frac{q_2 q_3}{r_{23}}
\]

\[
= \left(9.0 \times 10^9 \text{ N m}^2/\text{C}^2 \right) \left[ \frac{\left(2 \text{ nC}\right)(-1.0 \text{ nC})}{0.03 \text{ m}} + \frac{\left(2 \text{ nC}\right)(-1.0 \text{ nC})}{0.03 \text{ m}} + \frac{(-1 \text{ nC})(-1.0 \text{ nC})}{0.03 \text{ m}} \right]
\]

\[
= -0.60 \times 10^{-4} \text{ J} - 0.60 \times 10^{-6} \text{ J} + 0.30 \times 10^{-4} \text{ J} = -0.90 \times 10^{-4} \text{ J}
\]

Assess: Note that \(U_{12} = U_{21}, U_{13} = U_{31}, \) and \(U_{23} = U_{32} \).

29.7. Model: The charges are point charges.
Visualize: Please refer to Figure Ex29.7.
Solve: The electric potential energy of the electron is

\[
U_{\text{elec stren}} = U_{13} + U_{23}
\]

\[
= \left(9.0 \times 10^9 \text{ N m}^2/\text{C}^2 \right) \left[ \frac{\left(1.60 \times 10^{-19} \text{ C}\right)(-1.60 \times 10^{-19} \text{ C})}{\sqrt{(2.0 \times 10^{-9} \text{ m})^2 + (0.5 \times 10^{-9} \text{ m})^2}} + \frac{\left(1.60 \times 10^{-19} \text{ C}\right)(-1.60 \times 10^{-19} \text{ C})}{\sqrt{(2.0 \times 10^{-9} \text{ m})^2 + (0.5 \times 10^{-9} \text{ m})^2}} \right]
\]

\[
= -1.12 \times 10^{-19} \text{ J} - 1.12 \times 10^{-19} \text{ J} = -2.24 \times 10^{-19} \text{ J}
\]

29.8. Model: The electrons and the proton are point charges.
Visualize:

\[
q_1 = -e
\]
\[
q_2 = -e
\]
\[
q_3 = e
\]
\[
r_{12} = r_{23} = r_{13} = 1.0 \times 10^{-9} \text{ m}
\]

Solve: We are given that \(r_{12} = r_{23} = r_{13} = 1.0 \times 10^{-9} \text{ m}\). From the geometry of the figure,

\[
\frac{r_{23}}{r_{24}} = \cos 30^\circ \Rightarrow r_{23} = \frac{r_{24}}{2 \cos 30^\circ} = 0.5774 \times 10^{-9} \text{ m} = r_{14} = r_{34}
\]

The contributions to the total potential energy are

\[
U_{12} = U_{13} = U_{23} = \frac{\left(9.0 \times 10^9 \text{ N m}^2/\text{C}^2 \right)\left(1.60 \times 10^{-19} \text{ C}\right)(-1.60 \times 10^{-19} \text{ C})}{1.0 \times 10^{-9} \text{ m}} = 2.30 \times 10^{-10} \text{ J}
\]

\[
U_{14} = U_{24} = U_{34} = \frac{\left(9.0 \times 10^9 \text{ N m}^2/\text{C}^2 \right)\left(1.60 \times 10^{-19} \text{ C}\right)(1.60 \times 10^{-19} \text{ C})}{0.5774 \times 10^{-9} \text{ m}} = -3.99 \times 10^{-10} \text{ J}
\]

Summing all of the contributions,

\[
U_{\text{elec stren}} = U_{12} + U_{13} + U_{23} + U_{14} + U_{24} + U_{34}
\]

\[
= 2 \left(2.30 \times 10^{-10} \text{ J} \right) + 3 \left(-3.99 \times 10^{-10} \text{ J} \right) = -5.06 \times 10^{-10} \text{ J}
\]

Assess: Note that \(U_{12} = U_{21}, \ U_{13} = U_{31}, \) and \(U_{23} = U_{32}, U_{14} = U_{41}, U_{24} = U_{42}, \) and \(U_{34} = U_{43}.\)
29.9. **Model:** A water molecule is at the point of minimum potential energy when it is aligned with an electric field. However, an external force can rotate the water molecule causing its dipole moment to make an angle with the field.

**Solve:** The potential energy of an electric dipole moment in a uniform electric field is given by Equation 29.23:

\[ U_{\text{dipole}} = -\vec{p} \cdot \vec{E} = -pE \cos \theta. \]

This means

\[ U_{\text{dipole parallel}} = -pE \cos 0 = -pE \]
\[ U_{\text{dipole perpendicular}} = -pE \cos 90 = 0 \text{ J} \]

Hence,

\[ \Rightarrow U_{\text{dipole parallel}} - U_{\text{dipole perpendicular}} = 1.0 \times 10^{-21} \text{ J} = 0 \text{ J} - (-pE) \]

\[ \Rightarrow E = \frac{1.0 \times 10^{-21}}{p} = \frac{1.0 \times 10^{-21} \text{ J}}{6.2 \times 10^{-19} \text{ C m}} = 1.61 \times 10^3 \text{ N/C} \]

**Assess:** Note that the units with \( E \) are J/C m. Because 1 J/C m = 1 N m/C m = 1 N/C, the units of \( E \) are N/C.

29.10. **Model:** An external electric field supplies energy to a dipole.

**Visualize:** On an energy diagram, the oscillation occurs between the points where the potential-energy curve crosses the total energy line.

**Solve:** (a) The potential energy of an electric dipole moment in a uniform electric field is given by Equation 29.23: \( U = -\vec{p} \cdot \vec{E} = -pE \cos \theta \). This means

\[ U_{\theta=60} = -pE = -2 \mu \text{J} \]
\[ U_{\theta=-60} = -pE \cos 60 = -\frac{1}{2} pE = -\frac{1}{2} (2\mu) = -1 \mu\text{J} \]

The mechanical energy \( E_{\text{mech}} = U + K \). We know that at \( \theta = 60^\circ \), \( K_{\theta=60} = 0 \text{ J} \). So,

\[ E_{\text{mech}} = U_{\theta=60} + K_{\theta=60} = -1 \mu \text{J} + 0 \text{ J} = -1 \mu\text{J} \]

(b) Conservation of mechanical energy gives

\[ U_{\theta=60} + K_{\theta=60} = U_{\theta=-60} + K_{\theta=-60} \Rightarrow -1 \mu \text{J} + 0 \text{ J} = -2 \mu \text{J} + K_{\theta=-60} \Rightarrow K_{\theta=-60} = 1 \mu\text{J} \]

29.11. **Model:** Mechanical energy is conserved. The potential energy is determined by the electric potential.

**Visualize:**

Before
\[ v_i = 0 \text{ m/s} \]

After
\[ v_f = 1000 \text{ V} \]

\[ \Delta V = V_f - V_i = 1000 \text{ V} \]

The figure shows a before-and-after pictorial representation of an electron moving through a potential difference of 1000 V. A negative charge speeds up as it moves into a region of higher potential \( (U \rightarrow K) \).
Solve: The potential energy of charge \( q \) is \( U = qV \). Conservation of energy, expressed in terms of the electric potential \( V \), is

\[ K_f + qV_i = K_i + qV_f \Rightarrow K_f = K_i + q(V_i - V_f) = K_i - q(V_i - V_f) \]

\[ \Rightarrow \frac{1}{2}mv_i^2 = 0 J - (-e)(\Delta V) \Rightarrow v_i = \sqrt{\frac{2e\Delta V}{m}} = \sqrt{\frac{2(1.6 \times 10^{-19} C)(1000 \text{ V})}{9.11 \times 10^{-31} \text{ kg}}} = 1.87 \times 10^7 \text{ m/s} \]

Assess: Note that the electric potential difference of 1000 V already existed in space due to other charges or sources. The electron of our problem has nothing to do with creating the potential.

29.12. Model: Energy is conserved. The potential energy is determined by the electric potential.

Visualize:

\[ v_i = 0 \text{ m/s} \]
\[ v_f = 1.0 \times 10^6 \text{ m/s} \]
\[ \Delta V = -1000 \text{ V} \]

The figure shows a before-and-after pictorial representation of a proton moving through a potential difference of -1000 V. A positive charge speeds up as it moves into a region of lower potential \((U \rightarrow K)\).

Solve: The potential energy of charge \( q \) is \( U = qV \). Conservation of energy, expressed in terms of the electric potential \( V \), is

\[ K_f + qV_i = K_i + qV_f \Rightarrow K_f = K_i + q(V_i - V_f) = 0 J - q\Delta V \]

\[ \Rightarrow \frac{1}{2}mv_i^2 = -q(-1000 \text{ V}) \Rightarrow v_i = \sqrt{\frac{2(1.60 \times 10^{-19} C)(1000 \text{ V})}{1.67 \times 10^{-27} \text{ kg}}} = 4.38 \times 10^5 \text{ m/s} \]

Assess: Note that the proton of our problem has nothing to do with creating the potential difference of -1000 V.

29.13. Model: Energy is conserved. The potential energy is determined by the electric potential.

Visualize:

\[ v_i = 0 \text{ m/s} \]
\[ v_f = 1.0 \times 10^6 \text{ m/s} \]
\[ \Delta V = V_f - V_i \]

The figure shows a before-and-after pictorial representation of a He\(^+\) ion moving through a potential difference. The ion’s initial speed is zero and its final speed is \( 1.0 \times 10^6 \text{ m/s} \). A positive charge speeds up as it moves into a region of lower potential \((U \rightarrow K)\).

Solve: The potential energy of charge \( q \) is \( U = qV \). Conservation of energy, expressed in terms of the electric potential \( V \), is

\[ K_i + qV_i = K_i + qV_f \Rightarrow K_i = K_i + q(V_i - V_f) = 0 J - q\Delta V \]

\[ \Rightarrow \Delta V = \frac{K_i - K_i}{q} = \frac{0 J - \frac{1}{2}mv_i^2}{q} = \frac{4(1.67 \times 10^{-27} \text{ kg})(1.0 \times 10^6 \text{ m/s})^2}{2(1.60 \times 10^{-19} \text{ C})} = -2.09 \times 10^4 \text{ V} \]

Assess: This result implies that the helium ion moves from a higher potential toward a lower potential.

29.14. Model: Energy is conserved. The potential energy is determined by the electric potential.

Visualize:

\[ v_i = 0 \text{ m/s} \]
\[ v_f = 1.0 \times 10^6 \text{ m/s} \]
\[ \Delta V = V_f - V_i \]

The figure shows a before-and-after pictorial representation of an electron moving through a potential difference. Because the negative electron gains speed as it travels, it moves into a region of higher potential \((U \rightarrow K)\).
Solve: The potential energy of charge \( q \) is \( U = qV \). Using the conservation of energy equation,

\[
K_i + qV_i = K_f + qV_f \Rightarrow V_f - V_i = \Delta V = \frac{1}{q} \left( K_f - K_i \right) = \frac{1}{(-e)} \left( 0 - \frac{1}{2}mv_i^2 \right)
\]

\[
\Rightarrow \Delta V = \frac{mv_i^2}{2e} = \frac{\left( 9.11 \times 10^{-31} \text{ kg} \right) \left( 1.0 \times 10^4 \text{ m/s} \right)^2}{2\left( 1.60 \times 10^{-19} \text{ C} \right)} = 2.85 \text{ V}
\]

Assess: A positive value of \( \Delta V \) shows that the electron moved from a region of lower potential to a region of higher potential.

29.15. Model: Energy is conserved. The potential energy is determined by the electric potential.

Visualize:

Before
\[
\begin{array}{c}
\rightarrow \quad \downarrow \\
V_i \\
\end{array}
\]

After
\[
\begin{array}{c}
\leftarrow \quad \uparrow \\
V_f \\
\end{array}
\]

The figure shows a before-and-after pictorial representation of an electron moving through a potential difference.

Solve: (a) Because the electron is a negative charge and it slows down as it travels, it must be moving from a region of higher potential to a region of lower potential.

(b) Using the conservation of energy equation,

\[
K_i + U_i = K_f + U_f \Rightarrow K_i + qV_i = K_f + qV_f
\]

\[
\Rightarrow V_f - V_i = \frac{1}{q} \left( K_f - K_i \right) = \frac{1}{(-e)} \left( \frac{1}{2}mv_i^2 - 0 \text{ J} \right)
\]

\[
\Rightarrow \Delta V = -\frac{mv_i^2}{2e} = \frac{-\left( 9.11 \times 10^{-31} \text{ kg} \right) \left( 5.0 \times 10^5 \text{ m/s} \right)^2}{2\left( 1.60 \times 10^{-19} \text{ C} \right)} = -0.712 \text{ V}
\]

Assess: The negative sign with \( \Delta V \) verifies that the electron moves into a higher potential region from a lower potential region.

29.16. Model: Energy is conserved. The potential energy is determined by the electric potential.

Visualize:

Before
\[
\begin{array}{c}
\rightarrow \quad \downarrow \\
V_i \\
\end{array}
\]

After
\[
\begin{array}{c}
\leftarrow \quad \uparrow \\
V_f \\
\end{array}
\]

The figure shows a before-and-after pictorial representation of a proton moving through a potential difference.

Solve: (a) Because the proton is a positive charge and it slows down as it travels, it must be moving from a region of lower potential to a region of higher potential.

(b) Using the conservation of energy equation,

\[
K_i + U_i = K_f + U_f \Rightarrow K_i + qV_i = K_f + qV_f
\]

\[
\Rightarrow V_f - V_i = \frac{1}{q} \left( K_f - K_i \right) = \frac{1}{(e)} \left( \frac{1}{2}mv_i^2 - 0 \text{ J} \right)
\]

\[
\Rightarrow \Delta V = \frac{mv_i^2}{2e} = \frac{\left( 1.67 \times 10^{-27} \text{ kg} \right) \left( 8.0 \times 10^5 \text{ m/s} \right)^2}{2\left( 1.60 \times 10^{-19} \text{ C} \right)} = 3340 \text{ V}
\]

Assess: A positive \( \Delta V \) confirms that the proton moves into a higher potential region.

29.17. Solve: By definition 1 V = 1 J/C and 1 J = 1 N m. Consequently,

\[
1 \frac{\text{V}}{\text{m}} = 1 \frac{\text{J}}{\text{C m}} = 1 \frac{\text{N m}}{\text{C m}} = 1 \frac{\text{N}}{\text{C}}
\]
29.18. Model: The electric potential difference between the plates is determined by the uniform electric field in the parallel-plate capacitor.

Solve: (a) The potential of an ordinary AA or AAA battery is 1.5 V. Actually, this is the potential difference between the two terminals of the battery. If the electric potential of the negative terminal is taken to be zero, then the positive terminal is at a potential of 1.5 V.

(b) If a battery with a potential difference of 1.5 V is connected to a parallel-plate capacitor, the potential difference between the two capacitor plates is also 1.5 V. Thus,

$$\Delta V_C = 1.5 \text{ V} = V_+ - V_- = Ed$$

where \( d \) is the separation between the two plates. The electric field inside a parallel-plate capacitor is

$$E = \frac{\eta}{\varepsilon_0} = \frac{Q}{A \varepsilon_0} \Rightarrow 1.5 \text{ V} = \left( \frac{Q}{A \varepsilon_0} \right) d$$

$$\Rightarrow Q = \left( \frac{1.5 \text{ V}}{A \varepsilon_0} \right) \frac{(1.5 \text{ V})(2.0 \times 10^{-2} \text{ m})}{2.0 \times 10^{-3} \text{ m}} = 8.34 \times 10^{-12} \text{ C}$$

Thus, the battery moves 8.34 \( \times \) \( 10^{-12} \) C of electron charge from the positive to the negative plate of the capacitor.

29.19. Model: The electric potential difference between the plates is determined by the uniform electric field in the parallel-plate capacitor.

Solve: (a) The potential difference \( \Delta V_C \) across a capacitor of spacing \( d \) is related to the electric field inside by

$$E = \frac{\Delta V_C}{d} \Rightarrow \Delta V_C = Ed = (1.0 \times 10^5 \text{ V/m})(0.002 \text{ m}) = 200 \text{ V}$$

(b) The electric field of a capacitor is related to the surface charge density by

$$E = \frac{\eta}{\varepsilon_0} = \frac{Q}{A \varepsilon_0}$$

$$\Rightarrow Q = \varepsilon_0 AE = (8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2)(4.0 \times 10^{-4} \text{ m}^2)(1.0 \times 10^5 \text{ V/m}) = 3.54 \times 10^{-10} \text{ C}$$

29.20. Model: The electric potential between the plates of a parallel-plate capacitor is determined by the uniform electric field between the plates.

Solve: (a) The potential difference across the plates of a capacitor is

$$\Delta V_C = Ed = \left( \frac{\eta}{\varepsilon_0} \right) d = \frac{Qd}{A \varepsilon_0} = \frac{0.708 \times 10^{-9} \text{ C})(1.0 \times 10^{-3} \text{ m})}{4.0 \times 10^{-4} \text{ m}^2}(8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2) = 200 \text{ V}$$

(b) For \( d = 2.0 \text{ mm} \), \( \Delta V_C = 400 \text{ V} \).

Assess: Note that the units in part (a) are N m/C. But Exercise 29.17 showed that 1 N/C = 1 V/m, so 1 N m/C = 1 V. We also see that the potential difference across a parallel-plate capacitor is directly proportional to the plate separation.

29.21. Model: The electric field inside a parallel-plate capacitor is determined by the potential difference between the plates. The proton's potential energy inside the capacitor is also determined by the capacitor's potential difference.

Visualize: Please refer to Figure Ex29.21.

Solve: (a) Because the right plate is at a higher potential compared with the left plate, the positive plate is on the right and has a potential of 300 V.

(b) The electric field strength inside the capacitor is

$$E = \frac{\Delta V_C}{d} = \frac{300 \text{ V} - 0 \text{ V}}{3.0 \times 10^{-3} \text{ m}} = 1.0 \times 10^5 \text{ V/m}$$

(c) The potential energy of a charge \( q \) is \( U = q \Delta V \). A proton on the left plate will have zero potential energy. A proton at the midpoint of the capacitor is at a potential of 150 V. Thus, its potential energy is

$$U = (1.6 \times 10^{-19} \text{ C})(150 \text{ V}) = 2.40 \times 10^{-17} \text{ J}$$

29.22. Model: The charge is a point charge.

Solve: (a) The electric potential of a charge \( q \) is

$$V = \frac{1}{4\pi \varepsilon_0} \frac{q}{r} \Rightarrow r = \frac{1}{4\pi \varepsilon_0} \frac{q}{V} = \frac{1}{4\pi \varepsilon_0} \frac{q}{(9.0 \times 10^9 \text{ Nm}^2/\text{C}^2)(25 \times 10^{-8} \text{ C})} \frac{225 \text{ Nm}^2/\text{C}}{V} = 225 \text{ Nm}^2/\text{C}$$
For $V = 1000$ V,

$$r_{1000} = \frac{225 \text{ N m}^2 / \text{C}}{1000 \text{ V}} = 0.225 \text{ m}$$

For $V = 2000$ V, $r_{2000} = 0.113 \text{ m}$; for $V = 3000$ V, $r_{3000} = 0.075 \text{ m}$; for $V = 4000$ V, $r_{4000} = 0.056 \text{ m}$.

(b) The potential of the ball bearing is the potential right on the surface of the ball bearing. Thus,

$$U_c = \left(-1.60 \times 10^{-19} \text{ C}\right)(900 \text{ V}) = -1.44 \times 10^{-16} \text{ J}$$

(c) The potential differences are

$$\Delta V_{AB} = V_B - V_A = 1800 \text{ V} - 1800 \text{ V} = 0 \text{ V} \quad \Delta V_{BC} = V_C - V_B = 900 \text{ V} - 1800 \text{ V} = -900 \text{ V}$$

29.24. Solve: Outside a charged sphere of radius $R$, the electric potential is identical to that of a point charge $Q$ at the center. That is,

$$V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r} \quad r \geq R$$

The potential of the ball bearing is the potential right on the surface of the ball bearing. Thus,

$$V = \frac{\left(9.0 \times 10^4 \text{ N m}^2 / \text{C}^2\right) \left(2.0 \times 10^9 \text{ C}\right) \left(-1.60 \times 10^{-19} \text{ C}\right)}{0.5 \times 10^{-3} \text{ m}} = -5760 \text{ V}$$
29.25. **Model:** Outside a charged sphere the electric potential is identical to that of a point charge at the center.

**Visualize:** Glass bead

**Solve:** For \( r \geq R \), \( V = \frac{Q}{4\pi \epsilon_0 r} \). The potentials at the two points are

\[
V_{2\text{mm}} = \left( \frac{9.0 \times 10^9 \text{ N m}^2 / \text{C}^2}{1 \text{ mm} + 2 \text{ mm}} \right) \left( \frac{9.0 \times 10^9 \text{ N m}^2 / \text{C}^2}{3.0 \times 10^{-3} \text{ m}} \right) \cdot \frac{1}{2} \text{ mm} \\
V_{4\text{mm}} = \left( \frac{9.0 \times 10^9 \text{ N m}^2 / \text{C}^2}{5.0 \times 10^{-3} \text{ m}} \right) \cdot \frac{1}{4} \text{ mm}
\]

\( \Rightarrow V_{2\text{mm}} - V_{4\text{mm}} = +500 \text{ V} \) \( \left( \frac{9.0 \times 10^9 \text{ N m}^2 / \text{C}^2}{3.0 \times 10^{-3} \text{ m}} \right) \left( \frac{1}{5.0 \times 10^{-3} \text{ m}} - \frac{1}{5.0 \times 10^{-3} \text{ m}} \right) \)

\( \Rightarrow Q = \frac{500 \text{ V}}{\left( \frac{15.0 \times 10^{-6} \text{ m}^2}{2.0 \times 10^{-3} \text{ m}} \right)} = 4.17 \times 10^{-10} \text{ C} \)

**Assess:** Do not forget to include the radius of the glass bead in \( r \).

29.26. **Model:** Outside a charged sphere the electric potential is identical to that of a point charge at the center.

**Solve:** (a) For a proton, assumed to be a point charge, the electric potential is

\[
V = \frac{1}{4\pi \epsilon_0} \frac{(+e)}{r} = \left( \frac{9.0 \times 10^9 \text{ N m}^2 / \text{C}^2}{0.053 \times 10^{-3} \text{ m}} \right) \frac{1.60 \times 10^{-19} \text{ C}}{0.053 \times 10^{-3} \text{ m}} = 27.2 \text{ V}
\]

(b) The potential energy of a charge \( q \) at a point where the potential is \( V \) is \( U = qV \). The potential energy of the electron in the proton’s potential is

\[
U = (-1.60 \times 10^{-19} \text{ C}) (27.2 \text{ V}) = -4.35 \times 10^{-18} \text{ J}
\]

29.27. **Model:** The net potential is the sum of the potentials due to each charge.

**Visualize:** Please refer to Figure Ex29.27.

**Solve:** The potential at the dot is

\[
V = \frac{1}{4\pi \epsilon_0} \frac{q_1}{r_1} + \frac{1}{4\pi \epsilon_0} \frac{q_2}{r_2} + \frac{1}{4\pi \epsilon_0} \frac{q_3}{r_3}
\]

\( = \left( \frac{9.0 \times 10^9 \text{ N m}^2 / \text{C}^2}{0.040 \text{ m}} \right) \left( \frac{2.0 \times 10^{-9} \text{ C}}{0.040 \text{ m}} + \frac{2.0 \times 10^{-9} \text{ C}}{0.050 \text{ m}} + \frac{2.0 \times 10^{-9} \text{ C}}{0.030 \text{ m}} \right) = +1410 \text{ V}
\]

**Assess:** Potential is a scalar quantity, so we found the net potential by adding three scalar quantities.

29.28. **Model:** The net potential is the sum of the potentials due to each charge.

**Visualize:** Please refer to Figure Ex29.28.

**Solve:** From the geometry in the figure,

\[
\frac{1.5 \text{ cm}}{r_1} = \frac{1.5 \text{ cm}}{r_2} = \frac{1.5 \text{ cm}}{r_3} = \cos 30^\circ \Rightarrow r_1 = r_2 = r_3 = \frac{1.5 \text{ cm}}{\cos 30^\circ} = 1.732 \text{ cm}
\]

The potential at the dot is

\[
V = \frac{1}{4\pi \epsilon_0} \frac{q_1}{r_1} + \frac{1}{4\pi \epsilon_0} \frac{q_2}{r_2} + \frac{1}{4\pi \epsilon_0} \frac{q_3}{r_3}
\]

\( = \left( \frac{9.0 \times 10^9 \text{ N m}^2 / \text{C}^2}{0.01732 \text{ m}} \right) \left( \frac{2.0 \times 10^{-9} \text{ C}}{0.01732 \text{ m}} - \frac{1.0 \times 10^{-9} \text{ C}}{0.01732 \text{ m}} - \frac{1.0 \times 10^{-9} \text{ C}}{0.01732 \text{ m}} \right) = 0 \text{ V}
\]

**Assess:** Potential is a scalar quantity, so we found the net potential by adding three scalar quantities.

29.29. **Model:** The net potential is the sum of the potentials due to each charge.

**Visualize:** Please refer to Figure Ex29.29.
The Electric Potential

29.11

Solve: (a) Let \( q_1 = +5 \text{ nC}, q_2 = -5 \text{ nC}, \) and \( q_3 = 10 \text{ nC}. \) Also, \( r_1 = 2 \text{ cm}, r_2 = 4 \text{ cm}, \) and \( r_3 = \sqrt{(2 \text{ cm})^2 + (4 \text{ cm})^2} = 4.47 \text{ cm}. \) Potential is a scalar, not a vector, so the net potential is simply the sum of the potentials of each of the charges. Each individual potential is simply that of a point charge, so

\[
V_A = V_1 + V_2 + V_3 = \frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2} + \frac{q_3}{4\pi\varepsilon_0 r_3} = \frac{1}{4\pi\varepsilon_0} \left( \frac{q_1 + q_2 + q_3}{r_1 + r_2 + r_3} \right)
\]

\[
= \left(9.0 \times 10^9 \text{ N m}^2 / \text{C}^2 \right) \left( \frac{5 \times 10^{-9} \text{ C}}{0.02 \text{ m}} + \frac{-5 \times 10^{-9} \text{ C}}{0.04 \text{ m}} + \frac{10 \times 10^{-9} \text{ C}}{0.0447 \text{ m}} \right) = 3140 \text{ V}
\]

(b) The potential energy of a proton at point \( A \) is

\[
U_{\text{proton}} = q_{\text{proton}} V_A = eV_A = (1.6 \times 10^{-19} \text{ C})(3140 \text{ V}) = 5.02 \times 10^{-16} \text{ J}
\]

Assess: Note that the units in part (a) were \( \text{N m/C}. \) But Problem 29.17 showed that \( 1 \text{ N/C} = 1 \text{ V/m}, \) so \( 1 \text{ N m/C} = 1 \text{ V}. \)

29.30. Model: The net potential is the sum of the potentials due to each charge.

Visualize: Please refer to Figure Ex29.30.

Solve: (a) Let \( q_1 = -5 \text{ nC}, q_2 = 10 \text{ nC}, \) and \( q_3 = 10 \text{ nC}. \) Also, \( r_1 = 2 \text{ cm}, r_2 = 4 \text{ cm}, \) and \( r_3 = \sqrt{(2 \text{ cm})^2 + (4 \text{ cm})^2} = 4.47 \text{ cm}. \) Each individual potential is simply that of a point charge, so

\[
V_B = V_1 + V_2 + V_3 = \frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2} + \frac{q_3}{4\pi\varepsilon_0 r_3} = \frac{1}{4\pi\varepsilon_0} \left( \frac{q_1 + q_2 + q_3}{r_1 + r_2 + r_3} \right)
\]

\[
= \left(9.0 \times 10^9 \text{ N m}^2 / \text{C}^2 \right) \left( \frac{-5 \times 10^{-9} \text{ C}}{0.02 \text{ m}} + \frac{10 \times 10^{-9} \text{ C}}{0.04 \text{ m}} + \frac{10 \times 10^{-9} \text{ C}}{0.0447 \text{ m}} \right) = 2010 \text{ V}
\]

(b) The potential energy of an electron at point \( B \) is

\[
U_{\text{electron}} = q_{\text{electron}} V_B = -eV_B = (-1.6 \times 10^{-19} \text{ C})(2010 \text{ V}) = -3.22 \times 10^{-16} \text{ J}
\]

Assess: Note that the units in part (a) are \( \text{N m/C}. \) But Problem 29.17 showed that \( 1 \text{ N/C} = 1 \text{ V/m}, \) so \( 1 \text{ N m/C} = 1 \text{ V}. \)

29.31. Model: The net potential is the sum of the scalar potentials due to each charge.

Visualize:

Solve: Let the point on the \( x \)-axis where the electric potential is zero be at a distance \( x \) from the origin. At this point, \( V_1 + V_2 = 0 \text{ V}. \) This means

\[
\frac{1}{4\pi\varepsilon_0} \left[ \frac{3.0 \times 10^{-9} \text{ C}}{x} + \frac{-1.0 \times 10^{-9} \text{ C}}{|x - 4.0 \text{ cm}|} \right] = 0 \text{ V} \Rightarrow -x + 3|x - 4.0 \text{ cm}| = 0 \text{ cm}
\]

Either \(-x + 3(x - 4.0 \text{ cm}) = 0 \text{ cm}, \) or \(-x + 3(4.0 \text{ cm} - x) = 0 \text{ cm}. \) In the first case, \( x = 6.0 \text{ cm} \) and, in the second case, \( x = 3 \text{ cm}. \) That is, we have two points on the \( x \)-axis where the potential is zero.

29.32. Model: The net potential is the sum of the scalar potentials due to each charge.

Visualize:
Solve: Let the point on the y-axis where the electric potential is zero be at a distance y from the origin. At this point, \( V_1 + V_2 = 0 \) V. This means

\[
\frac{1}{4\pi\varepsilon_0} \left( \frac{q_1}{r_1} + \frac{q_2}{r_2} \right) = 0 \quad \Rightarrow \quad \frac{-3.0 \times 10^{-6} \text{ C}}{\sqrt{(-9.0 \text{ cm})^2 + y^2}} + \frac{4.0 \times 10^{-6} \text{ C}}{\sqrt{(16.0 \text{ cm})^2 + y^2}} = 0
\]

\[
\Rightarrow 3\sqrt{(16 \text{ cm})^2 + y^2} = 4\sqrt{(-9 \text{ cm})^2 + y^2} \quad \Rightarrow \quad 9(256 \text{ cm}^2 + y^2) = 16(81 \text{ cm}^2 + y^2)
\]

\[
\Rightarrow 7y^2 = 1008 \text{ cm}^2 \quad \Rightarrow \quad y = \pm 12 \text{ cm}.
\]

29.33. Model: The potential is the sum of the scalar potentials due to each charge. But, the electric field is the vector sum of the fields due to each charge.

Visualize: Please refer to Figure Ex29.33.

Solve: (a) By the symmetry of the drawing about the middle, it appears that the magnitudes of the charges are the same. \( E_1 \) points left for \( x > b \) because \( E_1 \) is negative. So, the charge at \( x = b \) is negative. For \( x < a \), \( E_1 \) points left, so the charge at \( x = a \) is positive. For \( a < x < b \), \( E_1 \) is positive which is consistent with the charge choices.

(b)

The graph of the electric potential shown in the figure is consistent with the electric field as well as the charges.

29.34. Model: While the potential is the sum of the scalar potentials due to each charge, the electric field is the vector sum of the electric fields due to each charge.

Visualize: Please refer to Figure Ex29.34.

Solve: (a) By the symmetry of the drawing about the middle, we infer that the magnitudes of the charges are the same. As \( V \) is always positive, both charges must be positive. The positive signs for the two equal charges at \( x = a \) and at \( x = b \) are also consistent with the behavior of the potential in the range \( a < x < b \).

(b)

The graph of \( E_x \), the \( x \)-component of the electric field, as a function of \( x \) is shown in the figure.
29.35. **Model**: Consider the rod to be a line of charge.

**Visualize**: Divide the rod into small segments, each with charge ±Δq.

---

![Symmetrically placed segments](image)

**Solve**: Consider two segments, one positive and one negative, equally distant from the center of the rod. These segments are the same distance \( r \) from the dot. Thus the contribution of this pair of segments to the potential at the dot is

\[
V_+ + V_- = \frac{1}{4\pi\varepsilon_0} \frac{\Delta q}{r} - \frac{1}{4\pi\varepsilon_0} \frac{-\Delta q}{r} = 0 \text{ V}
\]

Since we can divide the entire rod into pairs of symmetrically placed segments, the net result of adding the potentials due to each pair is \( V = 0 \) V.

**Assess**: This conclusion depends on the dot being directly outward from the midpoint of the rod. The potential is *not* zero at other points.

29.36. **Solve**: Let the unknown charges be \( Q_1 \) and \( Q_2 \). Then \( Q_1 + Q_2 = 30 \times 10^{-6} \text{ C} \) and

\[
U = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{r_{12}} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{2.0 \times 10^{-2} \text{ m}} = -180 \times 10^{-6} \text{ J}
\]

\Rightarrow \quad Q_1 Q_2 = \frac{-(180 \times 10^{-6} \text{ J})(2.0 \times 10^{-2} \text{ m})}{9.0 \times 10^9 \text{ N m}^2/\text{C}^2} = -4.0 \times 10^{-16} \text{ C}^2

Solving the first equation for \( Q_2 \) and substituting into the second equation,

\[
Q_1 (30 \times 10^{-6} \text{ C} - Q_1) = -4.0 \times 10^{-16} \text{ C}^2 \Rightarrow Q_1 - (30 \times 10^{-6} \text{ C})Q_1 - (4.0 \times 10^{-16} \text{ C}^2) = 0
\]

\Rightarrow \quad Q_1 = \frac{(30 \times 10^{-6} \text{ C}) \pm \sqrt{(30 \times 10^{-6} \text{ C})^2 + 4(4.0 \times 10^{-16} \text{ C}^2)}}{2}

That is, the two charges are \(-10 \text{ nC} \) and \(40 \text{ nC} \).

**Assess**: As they must, the two charges when added yield a total charge of 30 nC, and when substituted into the potential energy equation yield \( U = -180 \times 10^{-6} \text{ J} \).

29.37. **Solve**: Let the two unknown, positive charges be \( Q_1 \) and \( Q_2 \). They are separated by a distance \( r_{12} = 5.0 \times 10^{-2} \text{ m} \). Their electric potential energy is

\[
U = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{r_{12}} = 72 \times 10^{-6} \text{ J} \Rightarrow \frac{72 \times 10^{-6} \text{ J}}{9.0 \times 10^9 \text{ N m}^2/\text{C}^2} = 4.0 \times 10^{-16} \text{ C}^2
\]

The electric force between these two charges is

\[
F = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{r_{12}^2} = \left(9.0 \times 10^9 \text{ N m}^2/\text{C}^2\right) \frac{4.0 \times 10^{-16} \text{ C}^2}{\left(5.0 \times 10^{-2} \text{ m}\right)^2} = 1.44 \times 10^{-3} \text{ N}
\]

**Assess**: As far as the magnitudes are concerned,

\[F = \frac{U}{r} = \frac{72 \times 10^{-6} \text{ J}}{5.0 \times 10^{-2} \text{ m}} = 1.44 \times 10^{-3} \text{ N}\]
29.38. **Model:** The charged beads are point charges.

**Visualize:**

**Before**

\[
q_A = -5 \text{nC} \\
q_B = -10 \text{nC} \\
m_A = 15 \text{g} \\
v_{iA} = 0 \text{m/s} \\
m_B = 25 \text{g} \\
v_{iB} = 0 \text{m/s}
\]

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**After**

\[
q_A = -5 \text{nC} \\
q_B = -10 \text{nC} \\
m_A = 15 \text{g} \\
v_{iA} = 0 \text{m/s} \\
m_B = 25 \text{g} \\
v_{iB} = 0 \text{m/s}
\]

\[
q_A = -5 \text{nC} \\
q_B = -10 \text{nC} \\
\text{Ininitely separated}
\]

**Solve:** Once the beads are released they will accelerate according to the force acting on them. The magnitude of the force between the beads is given by Coulomb's law:

\[
F_{AB} = \frac{1}{4\pi \varepsilon_0} \frac{|Q_A Q_B|}{r_{AB}^2} = \frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(5.0 \times 10^{-9} \text{ C})(10.0 \times 10^{-9} \text{ C})}{(0.12 \text{ m})^2} = 3.125 \times 10^{-5} \text{ N}
\]

The same force acts on bead A and bead B. Since \( F = ma \),

\[
a_A = \frac{3.125 \times 10^{-5} \text{ N}}{0.015 \text{ kg}} = 2.08 \times 10^{-3} \text{ m/s}^2 \\
a_B = \frac{3.125 \times 10^{-5} \text{ N}}{0.025 \text{ kg}} = 1.25 \times 10^{-3} \text{ m/s}^2
\]

where \( a_A \) is directed toward the left and \( a_B \) is directed toward the right. To find the maximum speeds \( v_{iA} \) and \( v_{iB} \) it is easier to use the conservation of momentum and conservation of energy equations than kinematics. The momentum conservation equation along x-direction \( p_{net} = p_{inert} = 0 \text{ kg m/s} \) means

\[-m_A v_{iA} + m_B v_{iB} = 0 \text{ kg m/s} \Rightarrow -(0.015 \text{ kg}) v_{iA} + (0.025 \text{ kg}) v_{iB} = 0 \text{ kg m/s} \Rightarrow v_{iB} = \frac{1}{5} v_{iA}\]

The conservation of energy conservation is \( U_i + K_i = U_f + K_f \). Noting that \( U_i \rightarrow 0 \text{ J as the masses are infinitely separated and \}

\[
U_i = \frac{1}{4\pi \varepsilon_0} \frac{Q_A Q_B}{r_{AB}} = \frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(5.0 \times 10^{-9} \text{ C})(10.0 \times 10^{-9} \text{ C})}{0.12 \text{ m}} = 3.75 \times 10^{-6} \text{ J}
\]

\[
\Rightarrow 0 \text{ J} + \left( \frac{1}{2} m_A v_{iA}^2 + \frac{1}{2} m_B v_{iB}^2 \right) = 3.75 \times 10^{-6} \text{ J} + 0 \text{ J}
\]

\[
\Rightarrow \frac{1}{2} (0.015 \text{ kg}) v_{iA}^2 + \frac{1}{2} (0.025 \text{ kg}) \left( \frac{1}{5} v_{iA} \right)^2 = 3.75 \times 10^{-6} \text{ J} \Rightarrow 0.012 v_{iA}^2 = 3.75 \times 10^{-6} \text{ J}
\]

Solving these equations yields \( v_{iA} = 1.77 \text{ cm/s} \) and \( v_{iB} = \frac{1}{5}(1.77 \text{ cm/s}) = 0.354 \text{ cm/s} \). These are the maximum speeds of the two beads and occur when they are infinitely separated from each other.
29.39. **Model:** While the net potential is the sum of the scalar potentials due to each charge, the net electric field is the vector sum of the electric fields.

**Visualize:**

```
\[ \vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 = 0 \text{ N/C} \implies E_1 = E_2 = \frac{1}{4\pi\varepsilon_0} \left( \frac{2 \times 10^{-9} \text{ C}}{(x+1.0 \text{ cm})^2} - \frac{2 \times 10^{-9} \text{ C}}{(x-1.0 \text{ cm})^2} \right) \]
```

Obviously, this equation can only be satisfied if \( x = \pm \infty \). That is, the electric field is zero at \( \pm \infty \) and at \( x = 0 \).

(b) Because potential is a scalar quantity, the potential of charge \( Q_1 \) is always negative and the potential of charge \( Q_2 \) is always positive. These two scalar numbers will add to give zero potential at \( x = 0 \) m and at \( \pm \infty \).

Mathematically this can be shown as follows. When \(-1.0 \text{ cm} \leq x \leq 1.0 \text{ cm} \), let the point of zero potential be at \( x \).

Then, the condition \( V_1 + V_2 = 0 \text{ V/m} \) is

```
\[ \frac{1}{4\pi\varepsilon_0} \left( -2 \times 10^{-9} \text{ C} \right) = \frac{1}{4\pi\varepsilon_0} \left( 2 \times 10^{-9} \text{ C} \right) \]
```

\( x = 0 \text{ cm} \).

When \( x > 1.0 \text{ cm} \),

```
\[ \frac{1}{4\pi\varepsilon_0} \left( -2 \times 10^{-9} \text{ C} \right) = \frac{1}{4\pi\varepsilon_0} \left( 2 \times 10^{-9} \text{ C} \right) \]
```

\( (x+1.0 \text{ cm}) = (x-1.0 \text{ cm}) \).

This equation can only be satisfied at \( x = \pm \infty \).

(c) The graphs are shown in the figure above.

29.40. **Model:** While the net potential is the sum of the potentials due to each charge, the net electric field is the vector sum of the electric fields.

**Visualize:**

```
\[ \vec{E}_1 = \frac{20.0 \text{ nC}}{4\pi\varepsilon_0} \quad \vec{E}_2 = \frac{-10.0 \text{ nC}}{4\pi\varepsilon_0} \]
```

The graphs are shown in the figure above.
The charge $Q_1 = 20.0 \text{ nC}$ is at the origin. The charge $Q_2 = -10.0 \text{ nC}$ is 15 cm to the right of the charge $Q_1$ on the $x$-axis.

Solve: (a) As the pictorial representation shows, the point $P$ on the $x$-axis where the electric field is zero can only be on the right side of the charge $Q_2$, that is, at $x \geq 15 \text{ cm}$. At this point $E_1 = E_2$, so we have

$$ \frac{1}{4 \pi \varepsilon_0} \frac{20.0 \times 10^{-9} \text{ C}}{x^2} = \frac{1}{4 \pi \varepsilon_0} \frac{10.0 \times 10^{-9} \text{ C}}{(x - 15.0 \text{ cm})^2} \Rightarrow x^2 = 2(x - 15.0 \text{ cm})^2 $$

$$ \Rightarrow x^2 - (60.0 \text{ cm})x + 450 \text{ cm}^2 = 0 \Rightarrow x = \frac{(60.0 \text{ cm}) \pm \sqrt{3600 \text{ cm}^2 - 1800 \text{ cm}^2}}{2} $$

$$ \Rightarrow x = 51.2 \text{ cm} \text{ and } 8.8 \text{ cm}. $$

The root $x = 8.8 \text{ cm}$ is not possible physically. So, the electric fields cancel out at $x = 51.2 \text{ cm}$. The electric potential at this point is

$$ V = \frac{1}{4 \pi \varepsilon_0} \frac{Q_1}{r_1} + \frac{1}{4 \pi \varepsilon_0} \frac{Q_2}{r_2} = \left(9.0 \times 10^9 \text{ N m}^2 / \text{C}^2\right) \left[\frac{20.0 \times 10^{-9} \text{ C}}{0.512 \text{ m}} + \frac{-10.0 \times 10^{-9} \text{ C}}{(0.512 \text{ m} - 0.150 \text{ m})}\right] = 103 \text{ V} $$

(b) The point on the $x$-axis where the potential is zero can be obtained from the condition $V_1 + V_2 = 0 \text{ V}$, which is

$$ \frac{1}{4 \pi \varepsilon_0} \frac{Q_1}{r_1} + \frac{1}{4 \pi \varepsilon_0} \frac{Q_2}{r_2} = 0 \Rightarrow \frac{20.0 \times 10^{-9} \text{ C}}{x} - \frac{10.0 \times 10^{-9} \text{ C}}{(15 \text{ cm} - x)} = 0 $$

$$ \Rightarrow (15 \text{ cm} - x) - x = 0 \Rightarrow x = 10 \text{ cm} $$

The electric field 10 cm away from charge $Q_1$, is

$$ \vec{E}_{\text{ext}} = \vec{E}_1 + \vec{E}_2 = \frac{1}{4 \pi \varepsilon_0} \frac{20.0 \times 10^{-9} \text{ C}}{(0.10 \text{ m})^2} \hat{i} + \frac{1}{4 \pi \varepsilon_0} \frac{(10.0 \times 10^{-9} \text{ C})}{(0.05 \text{ m})^2} \hat{i} = 5.40 \times 10^7 \hat{i} \text{ N/C} $$

29.41. Model: Mechanical energy is conserved. Metal spheres are point particles and they have point charges.

Visualize:

```
F_{B \rightarrow A}
A
q_A = 2 \mu C
m_A = 2 g

B
q_B = 2 \mu C
m_B = 4 g
```

Solve: (a) The system could have both kinetic and potential energy, although here $K = 0 \text{ J}$. The energy of the system is

$$ E_0 = K + U_0 = 0 + \frac{q_A q_B}{4 \pi \varepsilon_0 r_0} = \frac{(9.0 \times 10^9 \text{ N m}^2 / \text{C}^2)(2.0 \times 10^{-9} \text{ C})(2.0 \times 10^{-9} \text{ C})}{0.05 \text{ m}} = 0.720 \text{ J} $$

(b) In static equilibrium, the net force on sphere $A$ is zero. Thus

$$ T = F_{B \rightarrow A} = \frac{q_A q_B}{4 \pi \varepsilon_0 r_0} = \frac{(9.0 \times 10^9 \text{ N m}^2 / \text{C}^2)(2.0 \times 10^{-9} \text{ C})(2.0 \times 10^{-9} \text{ C})}{(0.05 \text{ m})^2} = 14.4 \text{ N} $$

(c) The spheres move apart due to the repulsive electric force between them. The surface is frictionless, so they continue to slide without stopping. When they are very far apart ($r_1 \rightarrow \infty$), their potential energy $U_1 \rightarrow 0 \text{ J}$. Energy is conserved, so we have

$$ E_i = K_i + U_i = \frac{1}{2} m_A v_{A1}^2 + \frac{1}{2} m_B v_{B1}^2 + 0 \text{ J} = E_0 $$

Momentum is also conserved: $P_{\text{after}} = m_A v_{A1} + m_B v_{B1} = P_{\text{before}} = 0 \text{ kg m/s}$. Note that these are velocities and that $v_{A1}$ is a negative number. From the momentum equation,

$$ v_{A1} = -\frac{m_B v_{B1}}{m_A} $$
Substituting this into the energy equation,

\[ E_0 = \frac{1}{2} m_A \left( \frac{m_B v_{Bi}^2}{m_A} \right) + \frac{1}{2} m_B v_{Bi}^2 = \frac{1}{2} m_A \left( \frac{m_B}{m_A} + 1 \right) v_{Bi}^2 \]

\[ \Rightarrow v_{Bi} = \sqrt{\frac{2E_0}{m_B \left( \frac{m_B}{m_A} + 1 \right)}} = \sqrt{\frac{2 \left( 0.720 \, \text{J} \right)}{(0.004 \, \text{kg})(4 \, g/2 \, g + 1)}} = 10.95 \, \text{m/s} \]

Using this result, we can then find \( v_{A1} = -\frac{m_B v_{Bi}}{m_A} = -21.9 \, \text{m/s} \). These are the velocities, so the final speeds are 21.9 m/s for the 2 g sphere and 10.95 m/s for the 4 g sphere.

29.42. Model: Apply the principles of conservation of energy and conservation of momentum.

Visualize:

Before

\[ m_p, v_p = 0.01c \]

Far apart

Alpha particle

After

\[ m_p, v_p = 0.01c \]

Proton

The figure shows a before-and-after pictorial representation of the charged particles moving toward each other. The proton's physical quantities are denoted by the subscript \( p \) and that of the alpha particle by the subscript \( a \).

Solve: The conservation of energy equation is \( K_f + U_f = K_i + U_i \). Initially when the charges are far away, \( U_i = 0 \, \text{J} \) and \( K_i = \frac{1}{2} m_p v_p^2 + \frac{1}{2} m_a v_a^2 \). At the distance of closest approach,

\[ U_f = \frac{1}{4\pi \varepsilon_0} \frac{(e)(2e)}{d} \quad K_f = \frac{1}{2} m_p v_p^2 + \frac{1}{2} m_a v_a^2 \]

The particles are not at rest at closest approach. That would violate conservation of momentum. Instead, the particles are instantaneously moving with the same velocity \( v_f = v_p = v_a \), so that the distance between them is (instantaneously) not changing. The conservation of energy equation simplifies to

\[ \frac{1}{2} \left( m_p + m_a \right) v_f^2 + \frac{1}{4\pi \varepsilon_0} \frac{2e^2}{d} = \frac{1}{2} \left( m_p + m_a \right) (0.01c)^2 \]

The conservation of momentum equation \( P_{slab} = P_{before} \) is

\[ m_p v_f + m_a v_f = m_p (0.01c) - m_a (0.01c) \]

\[ \Rightarrow v_f = \frac{(0.01c)(m_p - m_a)}{m_p + m_a} = \frac{(0.01)(3 \times 10^8 \, \text{m/s})(4u - 1u)}{4u + 1u} = 1.8 \times 10^6 \, \text{m/s} \]

Substituting into the energy-conservation equation,

\[ \frac{1}{2} (4u + 1u) (1.8 \times 10^6 \, \text{m/s})^2 + \frac{9.0 \times 10^9 \, \text{N m}^2 / \text{C}^2}{d} \frac{2(1.60 \times 10^{-19} \, \text{C})^2}{d} = \frac{4.608 \times 10^{-28} \, \text{N m}^2}{d} = \frac{1.44 \times 10^{13} \, \text{u m}^2 / \text{s}^2}{d} \]

\[ \Rightarrow d = \frac{4.608 \times 10^{-28} \, \text{N m}^2}{(1.44)(1.661 \times 10^{-27} \, \text{kg}) \times 10^{13} \, \text{m}^2 / \text{s}^2} = 1.93 \times 10^{-14} \, \text{m} \]

29.43. Model: Mechanical energy is conserved.

Visualize: Please refer to Figure P29.43. Label the 5.0 nC charge with subscript 1, the 3.0 nC with subscript 2, and so on.
Solve: The conservation of energy equation $K_i + U_i = K_f + U_f$ is

$$K_i + 0 J = 0 + U_i = K_i = U_i + U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$$

$$U_{12} = \frac{1}{4\pi \varepsilon_0} \frac{q_i q_2}{r_{12}} = \frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(5.0 \times 10^{-9} \text{ C})(3.0 \times 10^{-9} \text{ C})}{0.035 \text{ m}} = 3.857 \times 10^{-6} \text{ J}$$

$$U_{13} = \frac{1}{4\pi \varepsilon_0} \frac{q_i q_3}{r_{13}} = \frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(5.0 \times 10^{-9} \text{ C})(4.0 \times 10^{-9} \text{ C})}{\sqrt{(0.035 \text{ m})^2 + (0.015 \text{ m})^2}} = 4.727 \times 10^{-6} \text{ J}$$

$$U_{14} = \frac{1}{4\pi \varepsilon_0} \frac{q_i q_4}{r_{14}} = \frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(5.0 \times 10^{-9} \text{ C})(3.0 \times 10^{-9} \text{ C})}{0.015 \text{ m}} = 6.000 \times 10^{-6} \text{ J}$$

Likewise, $U_{23} = 7.200 \times 10^{-6} \text{ J}$, $U_{34} = 1.418 \times 10^{-6} \text{ J}$, and $U_{44} = 2.057 \times 10^{-6} \text{ J}$. The sum of all the potential energies is $25.3 \times 10^{-6} \text{ J}$, which is the final kinetic energy $K_f$.

29.44. Model: Energy is conserved. The potential energy is determined by the electric potential.

Visualize: Please refer to Figure P29.44.

Solve: The proton at point A is at a potential of $30 \text{ V}$ and its speed is $50,000 \text{ m/s}$. At point B, the proton is at a potential of $-10 \text{ V}$ and we are asked to find its speed. Clearly, the proton moves into a lower potential region, so its speed will increase. The conservation of energy equation $K_f + U_f = K_i + U_i$ is

$$K_i = \frac{1}{2} m v_i^2 + (+e)(-10 \text{ V}) = \frac{1}{2} m(50,000 \text{ m/s})^2 + (+e)(30 \text{ V})$$

$$\Rightarrow v_i = \sqrt{(50,000 \text{ m/s})^2 + \frac{2[(1.60 \times 10^{-19} \text{ C})(40 \text{ V})]}{1.67 \times 10^{-27} \text{ kg}}} = 1.01 \times 10^5 \text{ m/s}$$

Assess: The speed of the proton is higher, as expected.

29.45. Model: Energy is conserved. The electron’s potential energy inside the capacitor can be found from the capacitor’s electric potential.

Solve: (a) The voltage across the capacitor is

$$\Delta V_C = Ed = (5.0 \times 10^5 \text{ V/m})(2.0 \times 10^{-3} \text{ m}) = 1000 \text{ V}$$

(b) Because $E = \eta/\varepsilon_0$ for a parallel-plate capacitor, with $\eta = Q/A$, the charge on each plate is

$$Q = \pi R^2 E \varepsilon_0 = \pi(1.0 \times 10^{-2} \text{ m})^2(5.0 \times 10^5 \text{ V/m})(8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2) = 1.39 \times 10^{-9} \text{ C}$$

(c) The electron has charge $q = -e$, and its potential energy at a point where the capacitor’s potential is $V$ is $U = -eV$. Since the electron is launched from the negative (lower potential) plate toward the positive (higher potential) plate, its potential energy becomes more negative (because of the negative sign of the electron charge). That is, the potential energy decreases, which must lead to an increase in the kinetic energy. Conversely, the electron’s speed as it is launched is smaller than $2.0 \times 10^7 \text{ m/s}$. The conservation of energy equation is

$$K_i + qV_i = K_i + qV_i \Rightarrow \frac{1}{2} m v_i^2 = \frac{1}{2} m v_i^2 + q(V_i - V_i)$$

$$\Rightarrow v_i = v_i + \frac{2}{m}(-e)(1000 \text{ V})$$

$$\Rightarrow v_i = \sqrt{(2.0 \times 10^7 \text{ m/s})^2 - \frac{2[(1.60 \times 10^{-19} \text{ C})(1000 \text{ V})]}{9.11 \times 10^{-27} \text{ kg}}} = 7.0 \times 10^6 \text{ m/s}$$

29.46. Model: Energy is conserved.

Solve: (a) The conservation of energy equation $K_i + U_i = K_f + U_f$ is

$$K_i + qV_i = 0 J + qV_i \Rightarrow K_i = q(V_i - V_i)$$

The above equation can be rewritten separately for the proton and the electron as

$$K_{p0} = \frac{1}{2} m_p v_p^2 = (+e)(\Delta V_p) \quad K_{e0} = \frac{1}{2} m_e v_e^2 = (-e)(-\Delta V_e)$$
Note that we have written $V_p - V_i$ as $\Delta V_p$ for the proton, but $V_p - V_i = -\Delta V_i$ for the electron. This is because $V_p > V_i$ to speed up a proton, and $V_p > V_i$ to speed up an electron. With $v_p = v_i$, the above two equations give

$$\frac{\Delta V_p}{\Delta V_i} = \frac{m_p}{m_e} = \frac{1.67 \times 10^{-27} \text{ kg}}{9.11 \times 10^{-31} \text{ kg}} = 1833$$

(b) The first two equations of part (a) can be divided to give

$$\frac{K_{p_f}}{K_{e_f}} = \frac{\Delta V_p}{\Delta V_i}$$

For the same final kinetic energy of the proton and the electron $\Delta V_p/\Delta V_i = 1$.

29.47. **Model:** Energy is conserved.

**Solve:** (a)

The electric potential at the midpoint of the capacitor is 250 V. This is because the potential inside a parallel-plate capacitor is $V = Ed$ where $d$ is the distance from the negative electron. The proton has charge $q = e$ and its potential energy at a point where the capacitor’s potential is $V$ is $U = qV$. The proton will gain potential energy if it moves all the way to the positive plate. This increase in potential energy comes at the expense of kinetic energy which is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67 \times 10^{-27} \text{ kg})(200,000 \text{ m/s})^2 = 3.34 \times 10^{-17} \text{ J}$$

This available kinetic energy is not enough to provide for the increase in potential energy if the proton is to reach the positive plate. Thus the proton does not reach the plate because $K < \Delta U$.

(b) The energy-conservation equation $K_i + U_i = K_f + U_f$ is

$$\frac{1}{2}mv_i^2 + qV_i = \frac{1}{2}mv_f^2 + qV_i \Rightarrow \frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + q(V_f - V_i)$$

$$\Rightarrow v_f = \sqrt{v_i^2 + \frac{2q}{m}(V_f - V_i)} = \sqrt{(2.0 \times 10^8 \text{ m/s})^2 + \frac{2(1.60 \times 10^{-19} \text{ C})(250 \text{ V} - 0 \text{ V})}{1.67 \times 10^{-27} \text{ kg}}} = 2.96 \times 10^8 \text{ m/s}$$

29.48. **Model:** Energy is conserved. The proton’s potential energy inside the capacitor can be found from the capacitor’s potential difference.

**Visualize:** Please refer to Figure P29.48.

**Solve:** (a) The electric potential at the midpoint of the capacitor is 250 V. This is because the potential inside a parallel-plate capacitor is $V = Es$ where $s$ is the distance from the negative electron. The proton has charge $q = e$ and its potential energy at a point where the capacitor’s potential is $V$ is $U = qV$. The proton will gain potential energy if it moves all the way to the positive plate. This increase in potential energy comes at the expense of kinetic energy which is

$$\Delta U = e\Delta V = e(250 \text{ V}) = 1.60 \times 10^{-19} \text{ C}(250 \text{ V}) = 4.0 \times 10^{-17} \text{ J}$$

if it moves all the way to the positive plate. This increase in potential energy comes at the expense of kinetic energy which is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67 \times 10^{-27} \text{ kg})(200,000 \text{ m/s})^2 = 3.34 \times 10^{-17} \text{ J}$$

This available kinetic energy is not enough to provide for the increase in potential energy if the proton is to reach the positive plate. Thus the proton does not reach the plate because $K < \Delta U$.

(b) The energy-conservation equation $K_i + U_i = K_f + U_f$ is

$$\frac{1}{2}mv_i^2 + qV_i = \frac{1}{2}mv_f^2 + qV_f \Rightarrow \frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + q(V_f - V_i)$$

$$\Rightarrow v_f = \sqrt{v_i^2 + \frac{2q}{m}(V_f - V_i)} = \sqrt{(2.0 \times 10^8 \text{ m/s})^2 + \frac{2(1.60 \times 10^{-19} \text{ C})(250 \text{ V} - 0 \text{ V})}{1.67 \times 10^{-27} \text{ kg}}} = 2.96 \times 10^8 \text{ m/s}$$

29.49. **Model:** Energy is conserved. The proton ends up so far away from the two charges that we can consider its final potential energy to be zero.

**Visualize:** Please refer to Figure P29.49. The minimum speed to escape is the speed that allows the proton to reach $r_e = \infty$ when $v_i = 0 \text{ m/s}$. 
Solve: Both charges contribute equally to the proton’s initial potential energy, so we can calculate the potential energy once and multiply by two. The conservation of energy equation \( K_f + U_f = K_i + U_i \) is

\[
0 \text{ J} + 0 \text{ J} = \frac{1}{2}mv_i^2 + qV_i \Rightarrow 0 \text{ J} = \frac{1}{2}mv_i^2 + q \left[ 2 \frac{1}{4\pi\varepsilon_0} \left( \frac{-2 \text{nC}}{5.0 \times 10^{-3} \text{ m}} \right) \right]
\]

\[
= \frac{1}{2} \left[ 1.67 \times 10^{-27} \text{ kg} \right] v_i^2 = \frac{\left( 1.60 \times 10^{-19} \text{ C} \right) \left( 9.0 \times 10^9 \text{ N m}^2 / \text{C}^2 \right) \left( 2.0 \times 10^{-9} \text{ C} \right)}{5.0 \times 10^{-3} \text{ m}}
\]

\[
= v_i = 1.17 \times 10^6 \text{ m/s}
\]

29.50. Model: Energy is conserved. The electron ends up so far away from the plastic sphere that we can consider its potential energy to be zero.

Visualize:

The minimum speed to escape is the speed that allows the electron to reach \( r_f = \infty \) when \( v_f = 0 \text{ m/s} \).

Solve: The conservation of energy equation \( K_f + U_f = K_i + U_i \) is

\[
0 \text{ J} + 0 \text{ J} = \frac{1}{2}mv_i^2 + qV_i \Rightarrow 0 \text{ J} = \frac{1}{2}mv_i^2 + (-e) \left( \frac{1}{4\pi\varepsilon_0} \frac{1}{R} \right)
\]

\[
= v_i = \sqrt{\frac{2e}{m} \frac{1}{4\pi\varepsilon_0} \frac{q}{R}} = \sqrt{\frac{2 \left( 1.60 \times 10^{-19} \text{ C} \right)}{9.11 \times 10^{-31} \text{ kg}} \left( 9.0 \times 10^9 \text{ N m}^2 / \text{C}^2 \right) \left( 10 \times 10^{-9} \text{ C} \right) \left( 0.5 \times 10^{-2} \text{ m} \right)} = 7.95 \times 10^7 \text{ m/s}
\]

29.51. Model: Energy is conserved. The potential energy is determined by the electric potential.

Solve: The conservation of energy \( K_f + U_f = K_i + U_i \) is

\[
K_i + qV_i = K_i + qV_f \Rightarrow K_f = K_i + q(V_f - V_i)
\]

In this equation, \( K_i = \frac{1}{2}mv_i^2 = 0 \text{ J} \), the final proton potential is zero, and the initial proton potential is \( V \). We have \( K_f = eV \). That is, the maximum kinetic energy or maximum speed is directly proportional to the potential of the surface the proton is launched from. From Equation 29.36, Example 29.11, and Example 29.12, when \( z \to 0 \) for the ring and disk:

\[
V_{\text{sphere}} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R} \quad V_{\text{ring}} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R} \quad V_{\text{disk}} = \frac{1}{4\pi\varepsilon_0} \frac{2Q}{R}
\]

Because the potential on the disk is a factor of 2 larger than the potential on the sphere or the ring, the proton’s kinetic energy or the speed will be maximum in the case of the disk.

29.52. Model: Energy is conserved.

Visualize:

The dipole initially is at the higher potential energy. As it moves to its minimum energy state, the decrease in the potential energy appears as rotational kinetic energy.
Solve: The potential energy of a dipole is $U_{\text{dipole}} = -\mathbf{p} \cdot \mathbf{E} = -pE\cos\theta$, where $\theta$ is the angle between the dipole and the electric field. The energy conservation equation $K_i + U_i = K_f + U_f$ is

$$\frac{1}{2}mv^2 + U_i = 0 \Rightarrow \frac{1}{2}mv^2 = U_i - U_f = -pE\cos90^\circ - (-pE\cos\theta) \Rightarrow \frac{1}{2}mv^2 = pE \Rightarrow \omega = \sqrt{\frac{2pE}{I}}$$

The moment of inertia of the dipole is $I_{\text{dipole}} = 2m(\frac{1}{2}s)^2$. Substituting into the above equation,

$$\omega = \sqrt{\frac{2(2s)E}{2m(\frac{1}{2}s)^2}} = \sqrt{\frac{4(2.0 \times 10^{-9} \text{C})(0.10 \text{ m})(1000 \text{ V} / \text{ m})}{(1.0 \times 10^{-3} \text{ kg})(0.10 \text{ m})^2}} = 0.283 \text{ rad} / \text{s}$$

29.53. Model: Energy is conserved. Because the iron nucleus is very large compared to the proton, we will assume that the nucleus does not move (no recoil) and that the proton is essentially a point particle with no diameter.

Visualize: Before

$$r_i = \infty \quad \text{Fe} \quad q = +26 \text{ e} \quad v_i = 0$$

After

$$r_f = \text{Fe} \quad q = +26 \text{ e}$$

The proton is fired from a distance much greater than the nuclear diameter, so $r = \infty$ and $U_i = 0$ J. Because the nucleus is so small, a proton that is even a few atoms away is, for all practical purposes, at infinity. As the proton approaches the nucleus, it is slowed by the repulsive electric force. At the end point, the proton has just reached the surface of the nucleus ($r_i = \text{ nuclear diameter}$) with $v_f = 0$ m/s. (The proton won’t remain at this point but will be pushed back out again, but the subsequent motion is not part of this problem.)

Solve: Initially, the proton has kinetic energy but no potential energy. At the point of closest approach, where $v_f = 0$ m/s, the proton has potential energy but no kinetic energy. Energy is conserved, so $K_i + U_i = K_f + U_f$. This equation is

$$0 \text{ J} + \frac{(e)(26e)}{4\pi\varepsilon_0 r_i} = \frac{1}{2}m_{\text{proton}}v_i^2 + 0 \text{ J}$$

where $r_i$ is half the nuclear diameter. The initial speed of the proton is

$$v_i = \sqrt{\frac{2(e)(26e)}{4\pi\varepsilon_0 r_i m_{\text{proton}}}} = \sqrt{\frac{2(1.6 \times 10^{-19} \text{ C})(26 \times 1.6 \times 10^{-19} \text{ C})(9.0 \times 10^9 \text{ N m}^2 / \text{ C}^2)}{(1.67 \times 10^{-27} \text{ kg})(4.5 \times 10^{-15} \text{ m})}} = 3.99 \times 10^7 \text{ m} / \text{s}$$

29.54. Model: Energy is conserved. Because the mercury nucleus is very large compared to the proton, we will assume that the nucleus does not move (no recoil) and that the proton is essentially a point particle with no diameter.

Visualize: Before

$$r_i = \infty \quad \text{Hg} \quad q = 80 \text{ e} \quad v_i = 0 \text{ m/s}$$

After

$$d = \text{Hg} \quad q = 80 \text{ e}$$

The proton is fired from a distance much greater than the nuclear diameter, so $r = \infty$ and $U_i = 0$ J. Because the nucleus is so small, a proton that is even a few atoms away is, for all practical purposes, at infinity. As the proton approaches the nucleus, it is slowed by the repulsive electric force. At the end point of the problem, the proton reaches the distance of closest approach ($d$) with $v_f = 0$ m/s. (The proton won’t remain at this point but will be pushed back out again, but the subsequent motion is not part of the problem.)

Solve: Initially, the proton has kinetic energy but no potential energy. At the point of closest approach, where $v_f = 0$ m/s, the proton has potential energy but no kinetic energy. Energy is conserved, so $K_i + U_i = K_f + U_f$. This equation is

$$0 \text{ J} + \frac{e(80e)}{4\pi\varepsilon_0 d} = \frac{1}{2}m_{\text{proton}}v_i^2 + 0 \text{ J}$$

$$d = \frac{160e^2}{4\pi\varepsilon_0 m_{\text{proton}}v_i^2} = \frac{(9.0 \times 10^9 \text{ N m}^2 / \text{ C}^2)(160)(1.6 \times 10^{-19} \text{ C})^2}{(1.67 \times 10^{-27} \text{ kg})(4.0 \times 10^7 \text{ m} / \text{s})^2} = 1.38 \times 10^{-14} \text{ m} = 13.8 \text{ fm}$$

The radius of the nucleus is 7.0 fm, so the proton’s closest approach to the surface is 6.8 fm.
29.55. **Model:** Energy is conserved.

**Visualize:**

Before

\[ \text{Alpha particle, } q = 2e \]
\[ v_i\alpha = 0 \text{ m/s} \]

Thorium
\[ q = 90e \]

After

\[ \text{Alpha particle} \]

\[ v_f\alpha \]

\[ \text{Thorium} \]

The alpha particle is initially at rest \((v_i\alpha = 0 \text{ m/s})\) at the surface of the thorium nucleus. The potential energy of the alpha particle is \(U_{i\alpha}\). After the decay, the alpha particle is far away from the thorium nucleus, \(U_{f\alpha} = 0 \text{ J}\), and moving with speed \(v_{f\alpha}\).

**Solve:** Initially, the alpha particle has potential energy and no kinetic energy. As the alpha particle is detected in the laboratory, the alpha particle has kinetic energy but no potential energy. Energy is conserved, so \(K_{f\alpha} + U_{f\alpha} = K_{i\alpha} + U_{i\alpha}\). This equation is

\[
\frac{1}{2}mv_{f\alpha}^2 + 0 \text{ J} = 0 \text{ J} + \frac{1}{4\pi\epsilon_0} \frac{(2e)(90e)}{r_i}
\]

\[
\Rightarrow v_{f\alpha} = \sqrt{\frac{1}{4\pi\epsilon_0} \frac{360e^2}{mr_i}} = \sqrt{\frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(360)(1.60 \times 10^{-19} \text{ C})^2}{4(1.67 \times 10^{-27} \text{ kg})(7.5 \times 10^{-13} \text{ m})}} = 4.07 \times 10^7 \text{ m/s}
\]

29.56. **Model:** Energy is conserved.

**Visualize:**

\[ \text{Undergoes decay} \]

\[ \text{Before} \]

\[ 1 \text{ proton} \]

\[ \text{Hydrogen} \]

\[ q = +2e \]

\[ \text{Beta radiation} \]

\[ 2 \text{ neutrons} \]

\[ \text{Helium} \]

\[ \text{After} \]

\[ \text{Hydrogen} \]

\[ \text{Helium} \]

\[ \text{Beta radiation} \]

\[ v_i \rightarrow 0 \]

\[ r_i \rightarrow \infty \]

**Solve:**

(a) The neutron has zero charge. In the decay \(n \rightarrow p^+ + e^- + \nu\), where the neutrino \(\nu\) is chargeless, the net charge of the final state is \(e + (-e) + 0 = 0\). So charge is conserved.

(b) As shown in the figure, the tritium nucleus has one proton and two neutrons. It is the single proton that makes this an isotope of hydrogen, the \(Z = 1\) element in the periodic table. One of the neutrons undergoes the decay \(n \rightarrow p + e^- + \nu\). The electron and neutrino are ejected from the nucleus as beta radiation, but the new proton remains in the nucleus. The nucleus still has three nucleons, but now two are protons and only one is a neutron. An atom whose nucleus contains two protons is the \(Z = 2\) element of the periodic table, namely helium.

(c) As shown in the figure, the electron is "launched" from the surface of the \(^3\text{He}\) nucleus with speed \(v_i\). The initial energy

\[
E_i = K_i + U_i = \frac{1}{2} m_p v_i^2 + \frac{q_p q_e}{4\pi\epsilon_0 r_i} = \frac{1}{2} m_p v_i^2 + \frac{(-e)(2e)}{4\pi\epsilon_0 r_i}
\]

If the electron just barely escapes, it slows to \(v_f \rightarrow 0 \text{ m/s} \) as \(r_i \rightarrow \infty\). Its final energy is \(E_f = K_f + U_f = 0 \text{ J} + 0 \text{ J} = 0 \text{ J}\).

Energy is conserved, \(E_i = E_f\), so

\[
\frac{1}{2} m_p v_i^2 + \frac{(-e)(2e)}{4\pi\epsilon_0 r_i} = 0 \text{ J}
\]

\[
\Rightarrow v_i = \sqrt{\frac{2(-e)(2e)}{m_p 4\pi\epsilon_0 r_i}} = \sqrt{\frac{(2)(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(2)(1.60 \times 10^{-19} \text{ C})^2}{(9.11 \times 10^{-31} \text{ kg})(1.5 \times 10^{-13} \text{ m})}} = 8.21 \times 10^8 \text{ m/s}
\]

**Assess:** Our "classical" analysis gives an answer that exceeds the speed of light \((3 \times 10^8 \text{ m/s})\), which is forbidden by Einstein's theory of relativity. We would need to perform a relativistic analysis, which is beyond the scope of this chapter, to get the correct minimum escape speed.
29.57. **Model:** The electric field inside a capacitor is uniform.

**Solve:** (a) Because the parallel-plate capacitor was connected to the terminals of a 15 V battery for a long time, the potential difference across the capacitor right after the battery is disconnected is \(\Delta V = 15\) V. The electric field strength inside the capacitor is

\[
E = \frac{\Delta V_c}{d} = \frac{15\text{ V}}{0.5 \times 10^{-2}\text{ m}} = 3000\text{ V/m}
\]

Because \(E = \frac{\eta}{\varepsilon_0}\) for a parallel-plate capacitor and \(\eta = Q/A\), the total charge on each plate is

\[
Q = E\varepsilon_0 = (3000\text{ V/m})\pi(0.05\text{ m})^2(8.85 \times 10^{-12}\text{ C}^2/\text{Nm}^2) = 2.09 \times 10^{-10}\text{ C}
\]

(b) After the electrodes are pulled away to a separation of \(d' = 1.0\) cm, the charges on the plates are unchanged. That is, \(Q' = Q\). Because \(A' = A\), the electric field inside the capacitor is also unchanged. So, \(E' = E\). The potential difference across the capacitor is \(\Delta V'_c = E'd'\). Because \(d\) increases from 0.5 cm to 1 cm (\(d' = 1\) cm), the potential difference \(\Delta V'_c\) increases from 15 V to 30 V.

(c) When the electrodes are expanded, the new area is \(A' = \pi(r')^2 = \pi(2r)^2 = 4A\). The charge \(Q'\) on the capacitor plates, however, stays the same as before \((Q' = Q)\). The electric field is

\[
E' = \frac{\eta'}{\varepsilon_0} = \frac{Q'}{A'\varepsilon_0} = \frac{Q}{4A\varepsilon_0} = \frac{E}{4} = \frac{3000\text{ V/m}}{4} = 750\text{ V/m}
\]

The potential difference across the capacitor plates \(\Delta V'_c = E'd' = (750\text{ V})(0.05\text{ m}) = 3.75\text{ V}\).

29.58. **Model:** The electric field inside a capacitor is uniform.

**Solve:** (a) While the capacitor is attached to the battery, the plates are at the same potentials as the terminals of the battery. Thus, the potential difference across the capacitor is \(\Delta V_c = 15\) V. The electric field strength inside the capacitor is

\[
E = \frac{\Delta V_c}{d} = \frac{15\text{ V}}{0.5 \times 10^{-2}\text{ m}} = 3000\text{ V/m}
\]

Because \(E = \frac{\eta}{\varepsilon_0} = Q/A\varepsilon_0\), the charge on each plate is

\[
Q = E\varepsilon_0 = (3000\text{ V/m})\pi(0.05\text{ m})^2(8.85 \times 10^{-12}\text{ C}^2/\text{Nm}^2) = 2.09 \times 10^{-10}\text{ C}
\]

(b) After the electrodes are pulled away to a new separation of \(d' = 1.0\) cm, the potential difference across the capacitor stays the same as before. That is, \(\Delta V'_c = \Delta V_c = 15\) V. The electric field strength inside the capacitor is

\[
E' = \frac{\Delta V'_c}{d'} = \frac{15\text{ V}}{0.01\text{ m}} = 1500\text{ V/m}
\]

The charge on each plate is

\[
Q' = E'A\varepsilon_0 = (1500\text{ V}))(0.05\text{ m})^2(8.85 \times 10^{-12}\text{ C}^2/\text{Nm}^2) = 1.04 \times 10^{-10}\text{ C}
\]

(c) The potential difference \(\Delta V'_c = \Delta V_c = 15\) V is unchanged when the electrodes are expanded to area \(A' = 4A\). The electric field between the plates is

\[
E' = E = \frac{\Delta V'_c}{d'} = \frac{15\text{ V}}{0.5 \times 10^{-2}\text{ m}} = 3000\text{ V/m}
\]

The new charge is

\[
Q' = E'A\varepsilon_0 = 4E\varepsilon_0 = 4Q = 8.34 \times 10^{-12}\text{ C}
\]

29.59. **Solve:** (a) Outside a uniformly charged sphere, the electric field is identical to that of a point charge \(Q\) at the center. For \(r \geq R\),

\[
E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}
\]

Evaluating this at \(r = R\) and using Equation 29.36,

\[
E_0 = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2} = \frac{V_0}{R}
\]

(b) The field strength is

\[
E_0 = \frac{500\text{ V}}{0.5 \times 10^{-2}\text{ m}} = 100,000\text{ V/m}
\]
29.60. **Visualize:**

\[ V_A = 0.1 \text{ nC} \quad V_B = 0.1 \text{ nC} \]

\[ (V_{0A}) = 300 \text{ V} \quad (V_{0B}) = 300 \text{ V} \]

Solve: A sphere of radius \( R \) and charge \( Q \) has a surface potential \( V_s = Q/(4\pi \varepsilon_0 R) \). When the two drops A and B merge to form drop C, two quantities are conserved: the total charge and the total volume of mercury. The conservation of charge yields

\[ Q_C = Q_{\text{total}} = Q_A + Q_B = 0.1 \text{ nC} + 0.1 \text{ nC} = 0.2 \text{ nC} \]

Since the quantity of mercury does not change, the volumes of drops A, B, and C are related by

\[ \text{Vol}_C = \frac{4}{3} \pi R_C^3 \]

\[ \text{Vol}_A + \text{Vol}_B = \frac{4}{3} \pi R_A^3 + \frac{4}{3} \pi R_B^3 \]

We're not given the radii of drops A and B, but we know their surface potentials \((V_{0A})\) and \((V_{0B})\). Thus,

\[ (V_{0A}) = \frac{Q_A}{4\pi \varepsilon_0 R_A} \Rightarrow R_A = \frac{Q_A}{4\pi \varepsilon_0 (V_{0A})} = \frac{0.1 \times 10^{-9} \text{ C} (9.0 \times 10^9 \text{ N m}^2 / \text{C}^2)}{300 \text{ V}} = 3.00 \times 10^{-3} \text{ m} = R_B \]

Combining these radii gives \( R_C = 3.78 \times 10^{-3} \text{ m} \), and

\[ (V_{0C}) = \frac{Q_C}{4\pi \varepsilon_0 R_C} = \frac{(0.2 \times 10^{-9} \text{ C} (9.0 \times 10^9 \text{ N m}^2 / \text{C}^2)}{3.78 \times 10^{-3} \text{ m}} = 476 \text{ V} \]

29.61. **Solve:**

(a) Because the excess charge resides on the outer surface of a conductor, the charge placed on the inside surface of a hollow metal sphere will move rapidly to the outside surface.

(b) Because a spherical shell of charge has the same electric potential as a point charge \( Q \) at the center, the potential on the surface is

\[ V = \frac{1}{4\pi \varepsilon_0} \frac{Q}{R} \Rightarrow 500,000 \text{ V} = \left(9.0 \times 10^9 \text{ N m}^2 / \text{C}^2\right) \left(\frac{Q}{0.15 \text{ m}}\right) \Rightarrow Q = 8.33 \mu\text{C} \]

(c) The electric field inside a conductor is zero, so the electric field just inside the sphere is zero. From Gauss's Law, the electric field outside a charged conductor is

\[ E = \frac{\eta}{\varepsilon_0} = \frac{Q}{4\pi \varepsilon_0 (0.15 \text{ m})^2 (8.85 \times 10^{-12} \text{ C}^2 / \text{Nm}^2)} = 3.33 \times 10^6 \text{ V / m} \]

29.62. **Solve:**

A charged particle placed inside a uniformly charged spherical shell experiences no electric force. That is, \( E = 0 \text{ V/m} \) inside the shell. We know from Section 29.5 that a difference in potential between two points or two plates is the source of an electric field. Since the potential on the surface of the shell is \( V = Q/(4\pi \varepsilon_0 R) \), the potential inside must be the same. This ensures that the potential difference is zero and hence the electric field is zero inside the shell. The potential at the center of the spherical shell is thus the same as at the surface. That is, \( V_{\text{center}} = V_{\text{surface}} = Q/(4\pi \varepsilon_0 R) \).

29.63. **Model:**

The potential at any point is the superposition of the potentials due to all charges. Outside a uniformly charged sphere, the electric potential is identical to that of a point charge \( Q \) at the center.

**Visualize:** Please refer to Figure P29.63. Sphere A is the sphere on the left and sphere B is the one on the right.

**Solve:** The potential at point a is the sum of the potentials due to the spheres A and B:

\[ V_a = V_{A,AA} + V_{B,AA} = \frac{1}{4\pi \varepsilon_0} \frac{Q_A}{R_A} + \frac{1}{4\pi \varepsilon_0} \frac{Q_B}{0.70 \text{ m}} \]

\[ = (9.0 \times 10^9 \text{ N m}^2 / \text{C}^2) \left(100 \times 10^{-6} \text{ C} / 0.30 \text{ m}\right) + (9.0 \times 10^9 \text{ N m}^2 / \text{C}^2) \left(25 \times 10^{-6} \text{ C} / 0.70 \text{ m}\right) \]

\[ = 3000 \text{ V} + 321 \text{ V} = 3321 \text{ V} \]
Similarly, the potential at point b is the sum of the potentials due to the spheres A and B:

\[ V_b = V_{b_{ab}} + V_{A_{ab}} = \frac{1}{4\pi\varepsilon_0} \frac{Q_b}{R_b} + \frac{1}{4\pi\varepsilon_0} \frac{Q_A}{0.95 \text{ m}} \]

\[ = \left( 9.0 \times 10^9 \text{ N m}^2 / \text{C}^2 \right) \left( \frac{25 \times 10^{-9} \text{ C}}{0.05 \text{ m}} + \frac{100 \times 10^{-9} \text{ C}}{0.95 \text{ m}} \right) \]

\[ = 4500 \text{ V} + 947 \text{ V} = 5447 \text{ V} \]

Thus, the potential at point b is higher than the potential at a. The difference in potential is

\[ V_b - V_a = 5447 \text{ V} - 3321 \text{ V} = 2126 \text{ V}. \]

**Assess:** $V_{A_{ab}} = 3000 \text{ V}$ and the sphere B has a potential of 225 V at point a. The spherical symmetry dictates that the potential on a sphere’s surface be the same everywhere. So, in calculating the potential at point a due to the sphere B we used the center-to-center separation of 1.0 m rather than a separation of 100 cm - 30 cm = 70 cm from the center of sphere B to the point a. The former choice leads to the same potential everywhere on the surface whereas the latter choice will lead to a distribution of potentials depending upon the location of the point a. Similar reasoning also applies to the potential at point b.

**29.64. Model:** The charges making the dipole are point charges. The potential is the sum of potentials from each charge.

**Visualize:**

![Diagram of a dipole with point P far away](image)

Point P is far away compared to the separation s of the dipole charges, that is, $y \gg s$.

**Solve:** (a) The potentials at P due to the positive and negative charges of the dipole are

\[ V_+ = \frac{1}{4\pi\varepsilon_0} \frac{q}{y-s/2} \quad V_- = \frac{1}{4\pi\varepsilon_0} \frac{(-q)}{y+s/2} \]

\[ \Rightarrow V_{\text{dipole}} = V_+ + V_- = \frac{q}{4\pi\varepsilon_0} \left( \frac{1}{y-s/2} - \frac{1}{y+s/2} \right) = \frac{q}{4\pi\varepsilon_0} \left( \frac{s}{y^2 - s^2/4} \right) = \frac{1}{4\pi\varepsilon_0} \frac{p}{y^3} \]

(b) The potential due to the dipole moment of water is

\[ V_{\text{dipole}} = \left( 9.0 \times 10^8 \text{ N m}^2 / \text{C}^2 \right) \left( 6.2 \times 10^{-70} \text{ C m} \right) = 0.056 \text{ V} \]
29.65. **Model:** The potential at a point $P$ on the $x$-axis is the sum of potentials from the two positive point charges.

Visualize:

![Diagram showing two positive charges and point $P$ on the $x$-axis](image)

Point $P$ is such that $|x| << s$.

**Solve:** (a) The potentials at $P$ due to the two positive charges are

\[
V_{\text{rep}} = \frac{1}{4\pi \varepsilon_0} \left( \frac{q}{s/2 - x} \right)
\]

\[
V_{\text{lat}} = \frac{1}{4\pi \varepsilon_0} \left( \frac{q}{s/2 + x} \right)
\]

\[
\Rightarrow V_p = V_{\text{rep}} + V_{\text{lat}}
\]

\[
= \frac{q}{4\pi \varepsilon_0} \left[ \frac{1}{s/2 - x} + \frac{1}{s/2 + x} \right]
\]

\[
= \frac{q}{4\pi \varepsilon_0} \left( \frac{s}{s^2/4 - x^2} \right)
\]

We can use the binomial approximation $(1 + u)^n = 1 + nu$ if $u << 1$ to write

\[
V_p = \frac{q}{4\pi \varepsilon_0} \left( \frac{s}{s^2/4 - x^2} \right) = \frac{q}{4\pi \varepsilon_0} \left( \frac{4s}{s^2} \right) \left( 1 - \frac{4x^2}{s^2} \right)^{-1}
\]

\[
= \frac{q}{4\pi \varepsilon_0} \left( 1 + \frac{4x^2}{s^2} \right) = \frac{q}{\pi \varepsilon_0 s} + \frac{4qx^2}{\pi \varepsilon_0 s^3}
\]

(b) The first term is a constant that does not affect the motion. Because of the quadratic dependence on $x$ in the second term, the motion is simple harmonic motion.

29.66. **Model:** The potential is the sum of potentials from each charge.

Visualize:

![Diagram showing potential as a function of $x$](image)

Point $P$ is located on the $x$-axis and the two positive charges are located on the $y$-axis. The separation between the two charges is $s$. 
Solve: (a) The potentials at P due to the two positive charges are

\[
V_{\text{top}} = \frac{1}{4\pi\varepsilon_0} \sqrt{y^2 + s^2/4} \quad V_{\text{bottom}} = \frac{1}{4\pi\varepsilon_0} \sqrt{x^2 + s^2/4}
\]

\[
\Rightarrow V_{x+y} = V_{\text{top}} + V_{\text{bottom}} = \frac{1}{4\pi\varepsilon_0} \frac{2q}{\sqrt{x^2 + s^2/4}} = \frac{2q}{4\pi\varepsilon_0 x} \sqrt{1 + s^2/4x^2}
\]

(b) The V-versus-x graph is shown in the figure. The potential due to a charge \(2q\) located at the origin is \(V_{2q} = 2q/4\pi\varepsilon_0 x\), and is also plotted. This expression differs from the expression obtained in part (a) by the factor \((1 + s^2/4x^2)^{-1}\). For \(x \gg s\), this factor becomes 1 and the two potentials become the same.

29.67. Model: The net potential is the sum of potentials from all the charges.

Visualize: Please refer to Figure P29.67. Point P at which we want the net potential due to the linear electric quadrupole is far away compared to the separation \(s\), that is, \(y \gg s\).

Solve: The net potential at P is

\[
V_{\text{net}} = V_1 + V_2 + V_3 = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{y-s} + \frac{1}{4\pi\varepsilon_0} \frac{q_2}{y} + \frac{1}{4\pi\varepsilon_0} \frac{q_3}{y+s}
\]

\[
= \frac{1}{4\pi\varepsilon_0} \left[ \frac{(q_1) + \frac{(q_2) + (q_3)}{y+s}}{y} - \frac{(q_2 + (q_3))}{y} + \frac{(q_1 + (q_2) + (q_3))}{y-s} \right]
\]

At distances \(y \gg s\),

\[
V_{\text{net}} = \frac{1}{4\pi\varepsilon_0} \frac{(2qs^2)}{y^3} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{y^3}
\]

where \(Q = 2qs^2\) is called the electric quadrupole moment of the charge distribution.

Assess: This charge distribution is in fact a combination of two dipoles. As seen above their effects do not completely cancel.

29.68. Model: The disk has uniform surface charge density \(\eta = Q/A\).

Visualize: Please refer to Figure 29.32.

Solve: The on-axis potential of a disk of radius \(R\) and charge \(Q\) was found in Example 29.12 to be

\[
V_{\text{disk}} = \frac{Q}{2\pi\varepsilon_0 R} \left[ \ln \left(1 + \frac{z^2}{R^2}\right) - \frac{z}{R} \right]
\]

We’re interested in the case where \(z \gg R\) so \(R/z << 1\). To make use of the binomial approximation, which relies on very small numbers, the expression in brackets can be written as

\[
\sqrt{1 + \left(\frac{R}{z}\right)^2} - \frac{z}{R} = \sqrt{1 + \left(\frac{R}{z}\right)^2} - \frac{z}{R} = \frac{z}{R} \left[1 + \left(\frac{R}{z}\right)^2\right]^{-1/2} - \frac{z}{R}
\]

The binomial approximation is \((1 + x)^{1/2} = 1 + \frac{1}{2}x\) when \(x << 1\). The expression becomes

\[
\frac{z}{R} \left[1 + \frac{1}{2} \left(\frac{R}{z}\right)^2\right] - \frac{z}{R} = \frac{z}{R} \left[1 + \frac{1}{2} \left(\frac{R}{z}\right)^2\right] = \frac{R}{2z}
\]

As a result, the potential of the disk becomes

\[
V_{\text{disk}} = \frac{Q}{2\pi\varepsilon_0 R} \frac{R}{2z} = \frac{Q}{4\pi\varepsilon_0 z}
\]

This is the potential of a point charge \(Q\) at distance \(r = z\) on the z-axis.

Assess: The disk appears as a point charge from far away. This is understandable and reasonable.
29.69. **Model:** Assume the thin rod is a line of charge with uniform linear charge density.  
**Visualize:** Please refer to Figure P29.69. The point $P$ is a distance $d$ from the origin. Divide the charged rod into $N$ small segments, each of length $\Delta x$ and with charge $\Delta q$. Segment $i$, located at position $x_i$, contributes a small amount of potential $V_i$ at point $P$.

**Solve:** The contribution of the $i$th segment is

$$V_i = \frac{\Delta q}{4\pi\varepsilon_0 r_i} = \frac{\Delta q}{4\pi\varepsilon_0 (d-x_i)} = \frac{Q\Delta x/L}{4\pi\varepsilon_0 (d-x_i)}$$

Where $\Delta q = \lambda \Delta x$ and the linear charge density is $\lambda = Q/L$. We are placing the point $P$ at a distance $d$ rather than $x$ from the origin to avoid confusion with $x_i$. The $V_i$ are now summed and the sum is converted to an integral giving

$$V = \frac{Q}{4\pi\varepsilon_0 L} \int_{-d}^{d} \frac{dx}{d-x} = \frac{Q}{4\pi\varepsilon_0 L} \left[ -\ln|d-x| \right]_{-d}^{d} = -\frac{Q}{4\pi\varepsilon_0 L} \ln\left( \frac{d+L/2}{d-L/2} \right)$$

Replacing $d$ with $x$, the potential due to a line charge of length $L$ at a distance $x$ along the axis is

$$V = \frac{Q}{4\pi\varepsilon_0 L} \ln\left( \frac{x+L/2}{x-L/2} \right)$$

29.70. **Model:** Assume the thin rod is a line of charge with uniform linear charge density.  
**Visualize:** Please refer to Figure P29.69. The point $P$ is located a distance $z$ from the origin. Divide the charged rod into $N$ small segments, each of length $\Delta x$ and with charge $\Delta Q$. Segment $i$, located at position $x_i$, contributes a small amount of potential $V_i$ at point $P$.

**Solve:** The contribution of the $i$th segment is

$$V_i = \frac{\Delta Q}{4\pi\varepsilon_0 r_i} = \frac{\Delta Q}{4\pi\varepsilon_0 \sqrt{x_i^2+z^2}}$$

Charge $\Delta Q$ is related to length $\Delta x$ through the linear charge density: $\Delta Q = \lambda \Delta x = (Q/L)\Delta x$. Thus

$$V_i = \frac{Q\Delta x}{4\pi\varepsilon_0 L\sqrt{x_i^2+z^2}}$$

Potential is a scalar, so we can find the net potential at $P$ by summing all the $V_i$:

$$V = \sum_i V_i = \frac{Q}{4\pi\varepsilon_0 L} \sum_i \frac{\Delta x}{\sqrt{x_i^2+z^2}}$$

If we now let $\Delta x \to dx$, the sum becomes an integral over the length of the rod, from $x_{min} = -L/2$ to $x_{max} = +L/2$:

$$V = \frac{Q}{4\pi\varepsilon_0 L} \int_{-L/2}^{L/2} \frac{dx}{\sqrt{x^2+z^2}} = \frac{Q}{4\pi\varepsilon_0 L} \left[ \ln\sqrt{x^2+z^2} + x \right]_{-L/2}^{L/2} = \frac{Q}{4\pi\varepsilon_0 L} \ln\left( \frac{\sqrt{z^2+(L/2)^2}+L/2}{\sqrt{z^2+(L/2)^2}-L/2} \right)$$

We can also get a simpler expression by using symmetry to integrate from 0 to $L/2$ and multiplying by 2. This gives

$$V = \frac{Q}{2\pi\varepsilon_0} \ln\left( \frac{L}{2z} + \sqrt{1 + \left( \frac{L}{2z} \right)^2} \right)$$

This expression can be shown to be equivalent to the one obtained earlier.

29.71. **Model:** Because the rod is thin, assume the charge lies along the semicircle of radius $R$.

**Visualize:** Please refer to Figure P29.71. The bent rod lies in the $xy$-plane with point $P$ as the center of the semicircle. Divide the semicircle into $N$ small segments of length $\Delta s$ and of charge $\Delta Q = (Q/\pi R)\Delta s$, each of which can be modeled as a point charge. The potential $V$ at $P$ is the sum of the potentials due to each segment of charge.

**Solve:** The total potential is

$$V = \sum_i V_i = \sum_i \frac{1}{4\pi\varepsilon_0} \frac{\Delta Q}{R} = \frac{1}{4\pi\varepsilon_0 R} \sum_i \left( \frac{Q}{\pi R} \Delta s \right) = \frac{1}{4\pi\varepsilon_0 \pi R} \sum \Delta \theta = \frac{1}{4\pi\varepsilon_0 \pi R} \sum \Delta \theta.$$
All of the terms come to the front of the summation because these quantities did not change as far as the summation is concerned. The summation does not have to convert to an integral because the sum of all the $\Delta \theta$ around the semicircle is $\pi$. Hence, the potential at the center of a charged semicircle is

$$V_{\text{center}} = \frac{1}{4\pi \varepsilon_0} \frac{Q}{\pi R} = \frac{1}{4\pi \varepsilon_0} \frac{Q}{R}$$

29.72. Model: The disk has a uniform surface charge density $\eta = Q/A = Q/\pi (R_o^2 - R_i^2)$.

Visualize: Please refer to Figure 29.31. Orient the disk in the xy-plane, with point P at distance $z$. Divide the disk into rings of equal width $\Delta r$. Ring $i$ has radius $r_i$ and charge $\Delta Q$.

Solve: Using the result of Example 29.11, we write the potential at distance $z$ of ring $i$ as

$$V_i = \frac{1}{4\pi \varepsilon_0} \frac{\Delta Q}{r_i^2 + z^2} \Rightarrow V = \sum_i V_i = \frac{1}{4\pi \varepsilon_0} \sum_i \frac{\Delta Q}{r_i^2 + z^2}$$

Noting that $\Delta Q = \eta \Delta A = \eta 2\pi r_i \Delta r$,

$$V = \frac{1}{4\pi \varepsilon_0} \eta 2\pi \sum_i \frac{r_i \Delta r}{\sqrt{r_i^2 + z^2}} = \frac{\eta}{2\varepsilon_0} \int \frac{r \, dr}{\sqrt{r^2 + z^2}} = \frac{\eta}{2\varepsilon_0} \left[ \sqrt{r^2 + z^2} \right]_R = \frac{\eta}{2\varepsilon_0} \left[ \sqrt{R_o^2 + z^2} - \sqrt{R_i^2 + z^2} \right]$$

$$= \frac{Q}{2\pi \varepsilon_0 (R_o^2 - R_i^2)} \left[ \sqrt{R_o^2 + z^2} - \sqrt{R_i^2 + z^2} \right]$$

In the limit $R_i \to 0$ m,

$$V = \frac{Q}{2\pi \varepsilon_0 R_o^2} \left[ \sqrt{R_o^2 + z^2} - z \right]$$

This is the same result obtained for a disk of charge in Example 29.12.

29.73. Solve: (a) A charge of 40 nC is separated into two charges which are placed 3.0 cm apart. The potential energy of the charge arrangement is 90 $\mu$J. What are the two charges?

(b) We have $q_1 q_2 = 3.0 \times 10^{-16}$ C and $q_1 + q_2 = 40 \times 10^{-9}$ C. Substituting into the first equation an expression for $q_1$ from the second equation,

$$(40 \times 10^{-9} \text{ C} - q_2) q_2 = 3.0 \times 10^{-16} \text{ C}^2 \Rightarrow q_2^2 - (40 \times 10^{-9} \text{ C}) q_2 + 3.0 \times 10^{-16} \text{ C}^2 = 0$$

The solutions to this equation are $q_2 = 30$ nC and 10 nC. This means the two charges are 10 nC and 30 nC.

29.74. Solve: (a) A proton is shot toward a 2.0 nC point charge with a speed of $2.5 \times 10^6$ m/s. What is the proton’s speed when it is 1.0 mm from the charge?

(b) Solving the equation yields $v_f = 1.67 \times 10^6$ m/s.

29.75. Solve: (a) Charges $\pm Q$ are placed on the plates of a parallel-plate capacitor. Each plate is 2.0 cm x 2.0 cm and they are 1.0 mm apart. Find the magnitude of the charge on each plate if the voltage across the capacitor is 100 V.

(b) Solving the equation yields $Q = 0.35$ nC.

29.76. Solve: (a) Two 3.0 nC charges are distance $d$ apart. At a point 3.0 cm from one charge, on the side opposite the other charge, the potential is 1200 V. Find the separation $d$ between the charges.

(b) The given equation simplifies to

$$900 \text{ N m}^2 / \text{C} + \frac{27 \text{ N m}^2 / \text{C}}{0.03 \text{ m} + d} = 1200 \text{ V} \Rightarrow \frac{27 \text{ N m}^2 / \text{C}}{0.03 \text{ m} + d} = 300 \text{ V} \Rightarrow d = 0.060 \text{ m} = 6.0 \text{ cm}$$
29.77. **Model:** The charges are point charges. The potential at a point is the sum of the potentials from all the charges.

**Visualize:**

![Equation of a circle](image)

The two charges are in the xy-plane.

**Solve:** Let the potential be zero at the point P(x, y). Then,

\[
V_p = \frac{1}{4\pi \varepsilon_0} \frac{Q_1}{\sqrt{x^2 + y^2}} + \frac{1}{4\pi \varepsilon_0} \frac{Q_2}{\sqrt{(4.0 \text{ cm} - x)^2 + y^2}} = 0 \text{ V}
\]

\[
\Rightarrow \frac{3}{\sqrt{x^2 + y^2}} = \frac{9}{\sqrt{(4 - x)^2 + y^2}} \Rightarrow 9x^2 + 9y^2 = 16 + x^2 + y^2 - 8x
\]

\[
\Rightarrow 8x^2 - 16 + 8y^2 = 0 \Rightarrow (x + \frac{1}{2})^2 + y^2 = \frac{1}{4} = \frac{R^2}{4}
\]

This is the equation of a circle, \(x^2 + y^2 = R^2\), whose center is at \(x = -\frac{1}{4} \text{ cm}, y = 0 \text{ cm}\) and whose radius is \(R = \sqrt{\frac{1}{4}} = \frac{1}{2} \text{ cm}\). All values of \((x, y)\) that satisfy this equation are the points with zero potential. This zero potential contour map is shown as a dotted line in the figure.

**Assess:** It is clear from the contour map that two zero-potential points lying on the x-axis are at \(x = 1.0 \text{ cm}\) and \(x = -2.0 \text{ cm}\). Each position is nearer to the smaller charge and away from the bigger charge, as it must be.

29.78. **Model:** Energy is conserved. The charged spheres are point charges.

**Visualize:** Please refer to Figure CP29.78. Label the spheres 1, 2, 3, and 4 in a clockwise manner, with the sphere in the upper left-hand corner being sphere 1.

**Solve:** The total potential energy of the four spheres before they are allowed to move is

\[
U_i = (U_{12} + U_{13} + U_{14} + U_{i4}) = 0 \text{ J}
\]

All the charges are identical, \(r_{12} = r_{34} = r_{41} = 1 \text{ cm}\), and \(r_{34} = r_{34} = \sqrt{2} \text{ cm}\). The initial potential energy is

\[
U_i = 4 \left( \frac{1}{4\pi \varepsilon_0} \frac{(10 \times 10^{-9} \text{ C})(10 \times 10^{-9} \text{ C})}{0.01 \text{ m}} \right) + 2 \left( \frac{1}{4\pi \varepsilon_0} \frac{(10 \times 10^{-9} \text{ C})(10 \times 10^{-9} \text{ C})}{0.01414 \text{ m}} \right) = 48.73 \times 10^{-3} \text{ J}
\]

Since all charges are at rest, \(K_i = 0 \text{ J}\). As the spheres are allowed to move away from one another and they are far apart, \(U_f = 0 \text{ J}\). The final kinetic energy is

\[
K_f = 4 \left( \frac{1}{2} m v_i^2 \right) = 2(1.0 \times 10^{-3} \text{ kg}) v_i^2
\]

From the energy conservation equation \(U_i + K_i = U_f + K_f\)

\[
2(1.0 \times 10^{-3} \text{ kg}) v_i^2 = 48.73 \times 10^{-3} \text{ J} \Rightarrow v_i = 0.494 \text{ m/s}
\]

29.79. **Model:** Energy and momentum are conserved.

**Visualize:** Please refer to Figure CP29.79. Let the two spheres have masses \(m_a\) and \(m_b\), and speeds \(v_a\) and \(v_b\) when they are very far apart.
Solve: The energy conservation equation \( K_f + U_f = K_i + U_i \) is

\[
\left( \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 \right) + 0 J = 0 J + \frac{1}{4\pi\varepsilon_0} \frac{q_A q_B}{r_{AB}}
\]

\[
\Rightarrow \left( 0.002 \text{ kg} \right) v_A^2 + \left( 0.001 \text{ kg} \right) v_B^2 = \left( 9.0 \times 10^9 \text{ N m}^2 / \text{ C}^2 \right) \left( \frac{2.0 \times 10^{-9} \text{ C} \cdot 1.0 \times 10^{-9} \text{ C}}{2.0 \times 10^{-5} \text{ m}} \right)
\]

\[
\Rightarrow v_A^2 + 0.5 v_B^2 = 0.0090 \text{ m}^2 / \text{s}^2
\]

To solve for \( v_A \) and \( v_B \), we need another equation relating \( v_A \) and \( v_B \). From the momentum conservation equation \( P_{\text{after}} = P_{\text{before}} \) we get

\[
m_A v_A + m_B v_B = 0 \text{ kg m/s} \Rightarrow (2.0 \text{ g}) v_A + (1.0 \text{ g}) v_B = 0 \text{ kg m/s} \Rightarrow v_B = -2 v_A
\]

Substituting this expression into the energy conservation equation,

\[
v_A^2 + 0.5(2v_A)^2 = 0.009 \text{ m}^2 / \text{s}^2 \Rightarrow 3v_A^2 = 0.009 \text{ m}^2 / \text{s}^2
\]

Solving these equations, we get \( v_A = -0.0548 \text{ m/s} \) and \( v_B = +0.110 \text{ m/s} \). Thus the speeds are 0.0548 m/s for A and 0.110 m/s for B.

29.80. Model: Energy and momentum are conserved.

Visualize: Please refer to Figure CP29.80. Let the two spheres have masses \( m_c \) and \( m_d \), and speeds \( v_c \) and \( v_d \) when the two spheres collide.

Solve: The energy conservation equation \( K_f + U_f = K_i + U_i \) is

\[
\left( \frac{1}{2} m_c v_c^2 + \frac{1}{2} m_d v_d^2 \right) + \left( \frac{1}{4\pi\varepsilon_0} \frac{q_c q_d}{0.002 \text{ m}} \right) = 0 J + \frac{1}{4\pi\varepsilon_0} \frac{q_c q_d}{0.010 \text{ m}}
\]

The momentum conservation equation \( P_{\text{after}} = P_{\text{before}} \) is

\[
m_c v_c + m_d v_d = 0 \text{ kg m/s} \Rightarrow v_c = -\left( \frac{2.0 \text{ g}}{1.0 \text{ g}} \right) v_d = -2 v_d
\]

Substituting this expression in the energy equation,

\[
\frac{1}{2} (0.001 \text{ kg}) 4v_d^2 + \frac{1}{2} (0.002 \text{ kg}) v_d^2 = \left( 9.0 \times 10^9 \text{ N m}^2 / \text{ C}^2 \right) \left( 2.0 \times 10^{-9} \text{ C} \cdot 1.0 \times 10^{-9} \text{ C} \right) \left[ \frac{1}{0.010 \text{ m}} - \frac{1}{0.002 \text{ m}} \right]
\]

\[
\Rightarrow (0.003 \text{ kg}) v_d^2 = 7.2 \times 10^{-6} \text{ N m}
\]

Solving these equations gives \( v_d = 0.049 \text{ m/s} = 4.9 \text{ cm/s} \) and hence \( v_c = -9.8 \text{ cm/s} \). The speed of C is +9.8 cm/s.

29.81. Model: The charges making the dipole are point charges. The potential is the sum of potentials from each charge.

Visualize:

Point P is located at a distance \( r \) from the center of the dipole and it makes an angle \( \theta \) with the dipole axis.
Solve: The potential at P due to the dipole is

\[ V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r_x} + \frac{1}{4\pi\epsilon_0} \frac{(-Q)}{r_y} = \frac{Q}{4\pi\epsilon_0} \left[ \frac{1}{\sqrt{x^2 + (y - s/2)^2}} - \frac{1}{\sqrt{x^2 + (y + s/2)^2}} \right] \]

\[ = \frac{Q}{4\pi\epsilon_0} \left[ \frac{1}{\sqrt{x^2 + y^2 - y_s + (s/2)^2}} - \frac{1}{\sqrt{x^2 + y^2 + y_s + (s/2)^2}} \right] \]

Because \( x, y \gg s, (s/2)^2 \) can be ignored compared with \( x^2, y^2 \) and \( y_s \). The potential is

\[ V = \frac{Q}{4\pi\epsilon_0} \left[ \frac{1}{\sqrt{r^2 - y_s}} - \frac{1}{\sqrt{r^2 + y_s}} \right] = \frac{Q}{4\pi\epsilon_0} \left[ \frac{1}{r} \left( \frac{1 - y_s}{r^2} \right)^{1/2} - \frac{1}{r} \left( 1 + y_s \right)^{-1/2} \right] \]

Using the binomial expansion,

\[ V = \frac{Q}{4\pi\epsilon_0} r \left[ \left( 1 + \frac{y_s}{2r^2} + \cdots \right) - \left( 1 - \frac{y_s}{2r^2} + \cdots \right) \right] = \frac{Q}{4\pi\epsilon_0} \frac{1}{2} \frac{y_s}{r^2} \]

\[ = \frac{1}{4\pi\epsilon_0} \frac{(Qs)}{r^3} = \frac{1}{4\pi\epsilon_0} \frac{(Qs) \cos \theta}{r^3} = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^3} \]

where \( p = Qs \) is the dipole moment of the dipole.

29.82. Model: The potential energy is determined by the electric potential.

Solve: (a) The electric field in the parallel-plate capacitor when the plates have charges \( \pm q \) is

\[ E = \frac{\eta}{A \epsilon_0} = \frac{q}{A \epsilon_0} \Rightarrow \Delta V_C = Ed = \frac{qd}{A \epsilon_0} \]

The increase in the potential energy \( dU \) due to moving charge \( dq \) through potential difference \( \Delta V_C \) is

\[ dU = dq \Delta V_C = \frac{qd dq}{A \epsilon_0} \]

(b) Integrating the expression for \( dU \) from \( q = 0 \) (uncharged) to \( q = Q \) (fully charged),

\[ U = \int dU = \int_0^Q \frac{qd}{A \epsilon_0} dq = \frac{d}{A \epsilon_0} \int_0^Q q dq = \frac{d}{A \epsilon_0} \left[ \frac{q^2}{2} \right]_0^Q = \frac{1}{2} \left( \frac{Qd}{A \epsilon_0} \right) Q = \frac{1}{2} Q \Delta V_C \]

where we have used the expression for \( \Delta V_C \) from part (a).

29.83. Model: The potential energy is determined by the electric potential.

Solve: (a) A sphere of radius \( R \) that is uniformly charged with a charge \( q \) is at a potential \( V_0 \) (see Equation 29.36):

\[ V_0 = \frac{1}{4\pi\epsilon_0} \frac{q}{R} \]

The increase in the potential energy \( dU \) when an infinitesimal amount of charge \( dq \) is moved from infinity (where the potential is zero) to the surface of the sphere (where the potential is \( V_0 \)) is

\[ dU = dq V_0 = \frac{1}{4\pi\epsilon_0} \frac{q}{R} dq \]

(b) Integrating the expression for \( U \) from \( q = 0 \) (uncharged) to \( q = Q \) (fully charged),

\[ U = \int dU = \int_0^Q \frac{1}{4\pi\epsilon_0} \frac{q}{R} dq = \frac{1}{4\pi\epsilon_0} \frac{1}{2} \int_0^Q q dq = \frac{1}{4\pi\epsilon_0} \frac{1}{2} Q^2 R \]

(c) The self-energy of a proton is

\[ U = \frac{1}{4\pi\epsilon_0} \frac{Q^2}{2R} = \left( 9.0 \times 10^9 \text{ N m}^2 / \text{C}^2 \right) \left( 1.60 \times 10^{-19} \text{ C} \right)^2 \left( 2 \times 10^{-15} \text{ m} \right) = 2.30 \times 10^{-13} \text{ J} \]
29.84. **Model:** Assume the cylinder is uniformly charged.

**Visualize:** Divide the cylinder into narrow rings of width $dz$, radius $R$, and charge $dq$.

**Ring with charge $dq$**

- Connect the cylinder into narrow rings of width $dz$, radius $R$, and charge $dq$.

**Solve:** The potential at the center is the sum of the potential of all the rings. The potential at distance $z$ on the axis of a ring of charge was found in Example 29.11 to be

$$V_{\text{ring}} = \frac{1}{4\pi\varepsilon_0} \frac{dq}{\sqrt{R^2 + z^2}}$$

The surface area of the ring is $dA = (2\pi R)dz$, so the amount of charge is $dq = \eta \, dA = (2\pi R \eta)dz$. The surface area of the cylinder is $A = (2\pi R)L$, so the surface charge density is $\eta = Q/(2\pi RL)$. Thus $dq = (Q/L)dz$ and the potential of the ring is

$$V_{\text{ring}} = \frac{Q/L}{4\pi\varepsilon_0} \frac{dz}{\sqrt{R^2 + z^2}}$$

The potential at the center of the cylinder is found by summing (i.e., integrating) the potential due to all rings. That is,

$$V_{\text{neg}} = \frac{Q/L}{4\pi\varepsilon_0} \int_{-L/2}^{L/2} \frac{dz}{\sqrt{R^2 + z^2}} = \frac{2Q/L}{4\pi\varepsilon_0} \left[ \ln\left( \frac{z + \sqrt{R^2 + z^2}}{R} \right) \right]_{-L/2}^{L/2}$$

$$= -\frac{2Q/L}{4\pi\varepsilon_0} \ln\left( \frac{L}{2R} + \sqrt{1 + \left( \frac{L}{2R} \right)^2} \right)$$