ELECTROMAGNETIC INDUCTION

33.1. Model: Assume the magnetic field is uniform.
Visualize: Please refer to Figure Ex33.1. Since a motional emf was developed the field must be perpendicular to \( \vec{v} \). The positive charges experienced a magnetic force to the left. By the right-hand rule the field must be out of the page so that \( \vec{v} \times \vec{B} \) is to the left.
Solve: This is a straightforward use of Equation 33.3. We have
\[
B = \frac{\varepsilon}{vl} = \frac{0.050 \text{ V}}{(5.0 \text{ m/s})(0.10 \text{ m})} = 0.10 \text{ T}
\]
Assess: This is reasonable. Laboratory fields are typically up to a few teslas in magnitude.

33.2. Visualize:

To develop a motional emf the magnetic field needs to be perpendicular to both, so let’s say its direction is into the page.
Solve: This is a straightforward use of Equation 33.3. We have
\[
v = \frac{\varepsilon}{iB} = \frac{1.0 \text{ V}}{(1.0 \text{ m})(5.0 \times 10^{-3} \text{ T})} = 2.0 \times 10^4 \text{ m/s}
\]
Assess: This is an unreasonable speed for a car. It’s unlikely you’ll ever develop a volt.

33.3. Visualize:

The wire is pulled with a constant force in a magnetic field. This results in a motional emf and produces a current in the circuit. From energy conservation, the mechanical power provided by the puller must appear as electrical power in the circuit.
Solve: (a) Using Equation 33.6,
\[
P = F_{\text{pull}}v \Rightarrow v = \frac{P}{F_{\text{pull}}} = \frac{4.0 \text{ W}}{1.0 \text{ N}} = 4.0 \text{ m/s}
\]
(b) Using Equation 33.6 again,

\[ P = \frac{v^2 l^2 B^2}{R} \Rightarrow B = \sqrt{\frac{RF_{\text{mag}}}{v_l^2}} = \sqrt{(0.20 \Omega)(1.0 \text{ N})} = 2.24 \text{ T} \]

Assess: This is a reasonable field for the circumstances given.

33.4. Model: Assume the field changes abruptly at the boundary between the two sections.

Visualize: Please refer to Figure Ex33.4. The directions of the fields are opposite, so some flux is positive and some negative. The total flux is the sum of the flux in the two regions.

Solve: The field is constant in each region so we will use Equation 33.10. Take \( \vec{A} \) to be into the page. Then, it is parallel to the field in the left region so the flux is positive, and it is opposite to the field in the right region so the flux is negative. The total flux is

\[ \Phi = \vec{A} \cdot \vec{B} = A_1 B_\| \cos \theta_1 + A_2 B_\| \cos \theta_2 \\
= (0.20 \text{ m})^2 (2.0 \text{ T}) - (0.20 \text{ m})^2 (1.0 \text{ T}) = 0.040 \text{ Wb} \]

Assess: The flux is positive because the areas are equal and the stronger field is parallel to the normal of the surface.

33.5. Model: Consider the solenoid to be long so the field is constant inside and zero outside.

Visualize: Please refer to Figure Ex33.5. The field of a solenoid is along the axis. The flux through the loop is only nonzero inside the solenoid. Since the loop completely surrounds the solenoid, the total flux through the loop will be the same in both the perpendicular and tilted cases.

Solve: The field is constant inside the solenoid so we will use Equation 33.10. Take \( \vec{A} \) to be in the same direction as the field. The magnetic flux is

\[ \Phi = \vec{A} \cdot \vec{B} = A_{\circ} B_{\circ} \cos \theta \]

When the loop is tilted the component of \( \vec{B} \) in the direction of \( \vec{A} \) is less, but the effective area of the loop surface through which the magnetic field lines cross is increased by the same factor.

33.6. Model: Assume the field strength is uniform over the loop.

Visualize: Please refer to Figure Ex32.6. According to Lenz's law, the induced current creates an induced field that opposes the change in flux.

Solve: The original field is into the page within the loop and is changing strength. The induced, counterclockwise current produces a field out of the page within the loop that is opposing the change. This implies that the original field must be increasing in strength so the flux into the loop is increasing.

33.7. Model: Assume that the magnet is a bar magnet with field lines pointing away from the north end.

Visualize: As the magnets move, if they create a change in the flux through the solenoid, there will be an induced current and corresponding field. According to Lenz's law, the induced current creates an induced field that opposes the change in flux.

Solve: (a) When magnet 1 is close to the solenoid there is flux to the left through the solenoid. As magnet 1 moves away there is less flux to the left. The induced current will oppose this change and produce an induced current and a corresponding flux to the left. By the right-hand rule, this corresponds to a current in the wire into the page at the top of the solenoid and out of the page at the bottom of the solenoid. So, the current will be right to left in the resistor.

(b) When magnet 2 is close to the solenoid the diverging field lines of the bar magnet produce a flux to the left in the left half of the solenoid and a flux to the right in the right half. Since the flux depends on the orientation of the loop, the flux on the two halves have opposite sign and the net flux is zero. Moving the magnet away changes the strength of the field and flux, but the total flux is still zero. Thus there is no induced current.

33.8. Model: Assume the plane of the loop is perpendicular to the field direction.

Visualize: Please refer to Figure Ex33.8. The flux is due to the field through the area of the triangle. Only the left half gives a contribution as the field strength is zero on the right half.

Solve: (a) The flux is \( \Phi = A \cdot B \). Take \( \vec{A} \) to be into the page, perpendicular to the triangle, and thus parallel to \( \vec{B} \). In this case \( \Phi = AB \) where \( A \) is the area of half of the triangle. This smaller triangle has a base of 10 cm and height \( 20 \sin 60^\circ \) cm = 17.32 cm. Thus,

\[ \Phi = AB = \frac{1}{2} (0.10 \text{ m})(0.1732 \text{ m}) \times 0.1 \text{ T} = 8.66 \times 10^{-4} \text{ Wb} \]
(b) The flux is directed into the loop. According to Lenz's law, the induced current will try to prevent the decrease of flux. To do this, the field of the induced current will have to point into this loop. This requires the induced current to flow clockwise.

33.9. Visualize: Please refer to Figure Ex33.9. The changing current in the solenoid produces a changing flux in the loop. By Lenz's law there will be an induced current and field to oppose the change in flux.

Solve: The current shown produces a field to the right inside the solenoid. So there is flux to the right through the surrounding loop. As the current in the solenoid decreases there is less field and less flux to the right through the loop. There is an induced current in the loop that will oppose the change by creating an induced field and flux to the right. This requires a clockwise current.

33.10. Model: Assume the field is uniform.

Visualize: Please refer to Figure Ex33.10. If the changing field produces a changing flux in the loop there will be a corresponding induced emf and current.

Solve: (a) The induced emf is \( \mathcal{E} = \frac{d\Phi}{dt} \) and the induced current is \( I = \frac{\mathcal{E}}{R} \). The field \( B \) is changing, but the area \( A \) is not. Take \( \hat{A} \) to be out of the page and parallel to \( \vec{B} \), so \( \Phi = AB \). Thus,

\[
\mathcal{E} = A \frac{dB}{dt} = \pi r^2 \left( 0.50 \text{ m} \right)^2 \left( 0.50 \text{ T/s} \right) = 3.93 \times 10^{-3} \text{ V}
\]

\[
I = \frac{\mathcal{E}}{R} = \frac{3.93 \times 10^{-3} \text{ V}}{0.1 \text{ \Omega}} = 39.3 \text{ mA}
\]

The field is increasing out of the page. To prevent the increase, the induced field needs to point into the page. Thus, the induced current must flow clockwise.

(b) As in part a, \( \mathcal{E} = A(dB/dt) = 3.93 \times 10^{-3} \text{ V} \) and \( I = 39.3 \text{ mA} \). Here the field is into the page and decreasing. To prevent the decrease, the induced field needs to point into the page. Thus the induced current must flow clockwise.

(c) Now \( \hat{A} \) (left or right) is perpendicular to \( \vec{B} \) and so \( \hat{A} \cdot \vec{B} = 0 \text{ Wb} \). That is, the field does not penetrate the plane of the loop. If \( \Phi = 0 \text{ Wb} \), then \( \mathcal{E} = \frac{d\Phi}{dt} = 0 \text{ V/m} \) and \( I = 0 \text{ mA} \). There is no induced current.

Assess: Note that the induced field opposes the change.

33.11. Model: Assume the field strength is changing at a constant rate.

Visualize:

\[
\begin{array}{cccccc}
\times & \times & \times & \times & \times & \times \\
\times & \times & \times & \times & & \\
\times & \times & \times & \times & \times & \times \\
\times & \times & \times & \times & \times & \times \\
\times & \times & \times & \times & \times & \times \\
\end{array}
\]

The changing field produces a changing flux in the coil and there will be a corresponding induced emf and current.

Solve: The induced emf of the coil is

\[
\mathcal{E} = N \frac{d\Phi}{dt} = N \frac{d(\hat{A} \cdot \vec{B})}{dt} = N A \frac{d\vec{B}}{dt} = \pi r^2 \frac{dB}{dt} \left( 0.10 \text{ T} \right) \left( 10 \times 10^{-7} \text{ s}^{-1} \right) = 3.14 \text{ V}
\]

where we've used the fact that \( \vec{B} \) is parallel to \( \hat{A} \).

Assess: This seems to be a reasonable emf as there are many turns.

33.12. Model: Assume the field is uniform.

Visualize: Please refer to Figure Ex33.12. The motion of the loop changes the flux through it. This results in an induced emf and current.
Solve: The induced emf is $\mathcal{E} = \frac{d\Phi}{dt}$ and the induced current is $I = \mathcal{E}/R$. The area $A$ is changing, but the field $B$ is not. Take $A$ as being out of the page and parallel to $\vec{B}$, so $\Phi = AB$ and $d\Phi/dt = B(dA/dt)$. The flux is through that portion of the loop where there is a field, that is, $A = lx$. The emf and current are

$$\mathcal{E} = B \frac{dA}{dt} = B \frac{d(lx)}{dt} = Blv = \left[0.20 \ T\right] \left(0.050 \ m\right) \left(50 \ m/s\right) = 0.50 \ V$$

$$I = \frac{\mathcal{E}}{R} = \frac{0.50 \ V}{0.10 \Omega} = 5.0 \ A$$

The field is out of the page. As the loop moves the flux increases because more of the loop area has field through it. To prevent the increase, the induced field needs to point into the page. Thus, the induced current flows clockwise.

Assess: This seems reasonable since there is rapid motion of the loop.

33.13. Model: Assume the field is uniform across the loop.

Visualize: Please refer to Figure Ex33.13. There is a current in the loop so there must be an emf that is due to a changing flux. With the loop fixed the area is constant so the change in flux must be due to a changing field strength.

Solve: The induced emf is $\mathcal{E} = \frac{d\Phi}{dt}$ and the induced current is $I = \mathcal{E}/R$. The $B$ field is changing, but the area $A$ is not. Take $A$ as being into the page and parallel to $\vec{B}$, so $\Phi = AB$ and $d\Phi/dt = A(dB/dt)$. We have

$$\mathcal{E} = \frac{d\Phi}{dt} = A \frac{dB}{dt} \Rightarrow \frac{dB}{dt} = \frac{IR}{A} = \frac{150 \times 10^{-3} A}{0.10 \ m} = 2.34 \ T/s$$

The original field and flux is into the page. The induced counterclockwise current produces an induced field and flux that is out of the page. Since the induced field opposes the change, the field must be increasing.

33.14. Model: Assume the inductance is that of a solenoid.

Visualize: The inductance of a solenoid depends on the number of turns, the length, and the cross-sectional area.

Solve: The length in terms of the number of turns in a layer and the wire diameter is $l = N \cdot d_{\text{wire}}$. The total number of turns is $N = 4N_l$. For a solenoid we have

$$L_{\text{sol}} = \frac{\mu_0 N^2 A}{l} \Rightarrow N = \sqrt{\frac{L_{\text{sol}} N}{\mu_0 A}} = \sqrt{\frac{(0.050 \ m) \left(100 \times 10^{-3} \ H\right)}{(4\pi \times 10^{-7} \ H/m)(0.0050 \ m)^2}} = 225$$

$$\Rightarrow N_l = \frac{1}{4} N = 56 \Rightarrow d_{\text{wire}} = \frac{l}{N_l} = \frac{0.050 \ m}{56} = 8.9 \times 10^{-4} \ m = 0.89 \ mm$$

33.15. Model: Assume the current drops uniformly.

Visualize: Assume that the current is flowing left to right as in the figure. The changing current produces a changing flux in the inductor and a corresponding emf is developed across the inductor.

Solve: For the inductor the potential difference is

$$\Delta V = -L \frac{dl}{dt} = -L \frac{N}{\Delta t} = -(10\times 10^{-3} \ H) (50 \ A - 150 \ A) \times 10^{-3} \times \frac{100 \ V}{10.0 \times 10^{-6} \ s} = 100 \ V$$

As the current decreases, so does the flux. The induced current will oppose the change and try to maintain the original flux. This means that the induced current will be in the same direction as the original current. This induced current carries positive charge carriers to the right, so the potential will increase in the direction of the current.

33.16. Model: Assume that the current changes uniformly.

Visualize: We want to increase the current without exceeding a maximum potential difference.
Solve: Since we want the minimum time, we will use the maximum potential difference:

\[ \Delta V = -L \frac{dI}{dt} = -L \frac{\Delta I}{\Delta t} \Rightarrow \Delta t = L \left| \frac{\Delta I}{\Delta V_{\text{max}}} \right| = \frac{(200 \times 10^{-3} \text{ H}) (3.0 \text{ A} - 1.0 \text{ A})}{400 \text{ V}} = 1.0 \text{ ms} \]

Assess: If we change the current in any shorter time the potential difference will exceed the limit.

33.17. **Visualize:** The solenoid has inductance and when a current flows there is energy stored in the magnetic field.

**Solve:** The inductance and energy of the solenoid are

\[ L_{\text{solenoid}} = \frac{\mu_0 N^2 A}{l} = \frac{(4\pi \times 10^{-7} \text{ H/m})(200)^2 \pi (0.015 \text{ m})^2}{0.12 \text{ m}} = 2.96 \times 10^{-4} \text{ H} \]

\[ U_L = \frac{1}{2} L I^2 = \frac{1}{2} (2.96 \times 10^{-4} \text{ H}) (0.80 \text{ A})^2 = 9.47 \times 10^{-3} \text{ J} \]

33.18. **Visualize:** Changing the variable capacitor in combination with a fixed inductor will change the resonant frequency of the **LC** circuit.

**Solve:** Since the resonant frequency depends on the inverse square root of the capacitance a lower capacitance will produce a higher frequency and vice versa. The maximum frequency is

\[ \omega_{\text{max}} = \sqrt{\frac{1}{LC_{\text{min}}} = \sqrt{\frac{1}{(2.0 \times 10^{-3} \text{ H})(100 \times 10^{-12} \text{ F})}} = 2.24 \times 10^6 \text{ rad/s} } \]

The corresponding minimum is \( \omega_{\text{min}} = 1.58 \times 10^6 \text{ rad/s} \). These are angular frequencies so we can use \( f = \omega / 2\pi \) to find \( f_{\text{min}} = 2.52 \times 10^5 \text{ Hz} \) and \( f_{\text{max}} = 3.57 \times 10^5 \text{ Hz} \).

33.19. **Visualize:** A capacitor in combination with an inductor will have a resonant frequency determined by the inductance and capacitance.

**Solve:** We know the frequency and the capacitance so we can find the inductance. We have

\[ \omega = \sqrt{\frac{1}{LC}} \Rightarrow L = \frac{1}{\omega^2 C} = \frac{1}{(2\pi \times 100 \times 10^{-6} \text{ Hz})(10 \times 10^{-12} \text{ F})} = 2.53 \times 10^{-7} \text{ H} = 0.253 \text{ \mu H} \]

33.20. **Visualize:** The rate at which the charges leave the capacitor will determine the current through the inductor.

**Solve:** The charge on the capacitor is \( Q = Q_0 \cos \omega t \) where \( Q_0 = CV \) is the maximum charge at \( t = 0 \) s. The current is given by \( i = \frac{dQ}{dt} = \omega Q_0 \sin \omega t \). Since the sine function oscillates between +1 and −1, the maximum current is

\[ i_{\text{max}} = \omega Q_0 = \left( \frac{1}{\sqrt{LC}} \right) CV = \Delta V \left( \frac{C}{L} \right) = (5.0 \text{ V}) \left( \frac{0.10 \times 10^{-6} \text{ F}}{1.0 \times 10^{-3} \text{ H}} \right) = 5.00 \times 10^{-2} \text{ A} = 50 \text{ mA} \]

33.21. **Visualize:** Please refer to Figure Ex33.21. This is a simple **LR** circuit if the resistors in parallel are treated as an equivalent resistor in series with the inductor.

**Solve:** We can find the equivalent resistance from the time constant since we know the inductance. We have

\[ \tau = \frac{L}{R_{\text{eq}}} \Rightarrow R_{\text{eq}} = \frac{L}{\tau} = \frac{3.6 \times 10^{-3} \text{ H}}{10 \times 10^{-6} \text{ s}} = 360 \text{ \Omega} \]

The equivalent resistance is the parallel addition of the unknown resistor \( R \) and 600 \( \Omega \). We have

\[ \frac{1}{R_{\text{eq}}} = \frac{1}{600 \text{ \Omega}} + \frac{1}{R} \Rightarrow R = \frac{600 \text{ \Omega}(360 \text{ \Omega})}{600 \text{ \Omega} - 360 \text{ \Omega}} = 900 \text{ \Omega} \]

33.22. **Visualize:** Please refer to Figure Ex33.22. This is a simple **LR** circuit if the two resistors in series are treated as an equivalent resistor in series with the inductor.
Chapter 33

Solve: The time constant of the equivalent circuit is
\[ \tau = \frac{L}{R_{eq}} = \frac{L}{R_1 + R_2} = \frac{50 \times 10^{-3} \text{ H}}{500 \ \Omega} = 1.00 \times 10^{-4} \text{ s} \]

If the current is initially at the maximum, then it will decay exponentially with time according to \( I(t) = I_0 e^{-\frac{t}{\tau}} \). The time \( t_{1/2} \) when the current drops to half the initial value is found as follows:
\[ \frac{1}{2} I_0 = I_0 e^{-\frac{t}{\tau}} \Rightarrow \ln\left(\frac{1}{2}\right) = -\frac{t}{\tau} \Rightarrow t = \tau \ln 2 \approx (1.00 \times 10^{-4} \text{ s})0.693 = 6.93 \times 10^{-5} \text{ s} \]

Assess: This is less than \( \tau \), as expected, since the time constant is the amount of time it takes to decay to 0.37 of the initial current and we are only going to 1/2. This result is quite general and is characteristic of all exponential decay phenomena.

33.23. Visualize: Please refer to Figure P33.23. To calculate the flux we need to consider the orientation of the normal of the surface relative to the magnetic field direction. We will consider the flux through the surface in the two parts corresponding to the two different directions of the surface normals.

Solve: The flux is
\[ \Phi = \Phi_{\text{top}} + \Phi_{\text{left}} = A_{\text{top}} \cdot \vec{B} + A_{\text{left}} \cdot \vec{B} = A_{\text{top}} B \cos 45^\circ + A_{\text{left}} B \cos 45^\circ \]
\[ = 2 \times (0.050 \text{ m} \times 0.10 \text{ m})(0.050 \text{ T})\cos 45^\circ = 3.54 \times 10^{-4} \text{ Wb} \]

33.24. Model: Assume the field is uniform inside the rectangular region and zero outside. Visualize: The flux measures how much of the field penetrates the chosen surface. We will break the surface up because the magnetic field has different strengths over different parts of the surface of the loop.

Solve: For convenience we choose the normal to the loop to be into the page so it is in the same direction as the magnetic field. The total flux is
\[ \Phi = \int \vec{B} \cdot d\vec{A} = \int \vec{B} \cdot d\vec{A} + \int \vec{B} \cdot d\vec{A} = \int B dA \cos \theta = BA_{\text{inside}} \]
\[ = Bab \]

33.25. Model: Assume the field is uniform in space though it is changing in time.

Visualize: The changing magnetic field strength produces a changing flux through the loop, and a corresponding induced emf and current.

Solve: (a) Since the field is perpendicular to the plane of the loop, \( \vec{A} \) is parallel to \( \vec{B} \) and \( \Phi = AB \). The emf is
\[ \mathcal{E} = \frac{d\Phi}{dt} = A \frac{dB}{dt} = (0.20 \text{ m})^2 \frac{4 - 4t}{1} \text{ T/s} = 0.16(1-t) \]

The magnetic field is increasing over the interval \( 0 < t < 1 \) s and is decreasing over the interval \( 1 < t < 2 \) s, so the induced emf and current must have opposite signs in the second half of the time interval. We arbitrarily choose the sign to be positive during the first half.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>( B ) (T)</th>
<th>( \mathcal{E} ) (volts)</th>
<th>( I ) (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.00</td>
<td>0.16</td>
<td>1.6</td>
</tr>
<tr>
<td>0.5</td>
<td>1.50</td>
<td>0.08</td>
<td>0.8</td>
</tr>
<tr>
<td>1.0</td>
<td>2.00</td>
<td>0.00</td>
<td>0.0</td>
</tr>
<tr>
<td>1.5</td>
<td>1.50</td>
<td>-0.08</td>
<td>-0.8</td>
</tr>
<tr>
<td>2.0</td>
<td>0.00</td>
<td>-0.16</td>
<td>-1.6</td>
</tr>
</tbody>
</table>

(b) To plot the field and current we look at the form of the equations as a function of time. The magnetic field strength is quadratic with a maximum at \( t = 1 \) s and vanishing at \( t = 0 \) s and \( t = 2 \) s. The current equation is linear and decreasing, starting at 1.6 A at \( t = 0 \) s and going through zero at \( t = 1 \) s.

Assess: Notice in the graph how \( I = 0 \) A at \( t = 1 \) s, the instant in time when \( B \) is a maximum, that is, when \( dB/dt = 0 \). At this point the flux is (instantaneously) not changing so the corresponding induced emf and current are zero.
33.26. **Model:** Assume the field is uniform in space though it is changing in time.  
**Visualize:** The changing magnetic field strength produces a changing flux through the coil, and a corresponding induced emf and current.  
**Solve:**

\[ B(T) \]

\[ E(t) = d\Phi \over dt = NA \left( \frac{dB}{dt} \right) = 20\pi(0.025 \text{ m})^2(0.020 + 0.020t) \text{ T/s} = 7.85 \times 10^{-4}(1 + t) \text{ V} \]

\[ I(t) = \frac{E(t)}{R} = \frac{7.85 \times 10^{-4}(1 + t)}{0.50\Omega} = 1.57 \times 10^{-3}(1 + t) \text{ A} \]

(b) Since the field is perpendicular to the plane of the coil, \( \vec{A} \) is parallel to \( \vec{B} \) and \( \Phi = AB \). The emf is

\[ E(t) = 6.28 \times 10^{-2}(1 + t) \text{ V} \]

\[ I(t) = \frac{E(t)}{R} = 6.28 \times 10^{-2}(1 + t) \text{ A} \]

Assess: The field is always increasing and increasing more rapidly with time so the induced current is greater at 10 s than at 5 s.

33.27. **Model:** Assume the field is uniform in space though it is changing in time.  
**Visualize:** The changing magnetic field strength produces a changing flux through the coil, and a corresponding induced emf and current.  
**Solve:**

\[ B(T) \]

\[ E(t) = d\Phi \over dt = NA \left( \frac{dB}{dt} \right) = 50\pi(0.020 \text{ m})^2(1 - \frac{1}{2}t) \text{ T/s} = 6.28 \times 10^{-2}(1 - \frac{1}{2}t) \text{ V} \]

\[ I(t) = \frac{E(t)}{R} = \frac{6.28 \times 10^{-2}(1 - \frac{1}{2}t)}{1.0\Omega} = 6.28 \times 10^{-2}(1 - \frac{1}{2}t) \text{ A} \]

(c) Using the expression for \( I(t) \) from part (b),

\[ I(1 \text{ s}) = 6.28 \times 10^{-2}(1 - \frac{1}{2} \times 1) \text{ A} = 3.14 \times 10^{-2} \text{ A} = 31.4 \text{ mA} \]

Also, \( I(2 \text{ s}) = 0.0 \text{ A} \) and \( I(3 \text{ s}) = -31.4 \text{ mA} \).
33.28. Model: Assume that $B$ changes uniformly with time. 

Visualize:

![Diagram](image)

The magnetic strength is changing so the flux is changing and this will create an induced emf. The magnetic field is at an angle $\theta = 60^\circ$ to the normal of the plane of the coils.

Solve: The flux for a single loop of the coil is $\Phi = \vec{A} \cdot \vec{B} = BA \cos \theta$. The radius and angle don’t change with time, but $B$ does. According to Faraday’s law, the induced emf is

$$\mathcal{E} = N \frac{d\Phi}{dt} = N A \cos \theta \frac{d(BA)}{dt} = N \pi r^2 \cos \theta \frac{\Delta B}{\Delta t}$$

$$= 100\pi(0.010 \text{ m})^2(\cos 60^\circ) \left| \frac{1.50 \text{ T} - 0.50 \text{ T}}{0.60 \text{ s}} \right| = 2.62 \times 10^{-2} \text{ V} = 26.2 \text{ mV}$$

33.29. Model: Assume that $B$ changes uniformly with time. 

Visualize:

![Diagram](image)

The magnetic strength is changing so the flux is changing and this will create an induced emf. The magnetic field is at an angle $\theta = 45^\circ$ to the normal to the plane of the coils.

Solve: The flux for a single loop of the coil is $\Phi = \vec{A} \cdot \vec{B} = BA \cos \theta$. The radius and angle don’t change with time, but $B$ does. According to Faraday’s law, the induced emf is

$$\mathcal{E} = N \frac{d\Phi}{dt} = N A \cos \theta \frac{d(BA)}{dt} = N \pi r^2 \cos \theta \frac{\Delta B}{\Delta t}$$

$$= 25\pi(0.050 \text{ m})^2(\cos 45^\circ) \left| \frac{0.20 \text{ T} - 0.80 \text{ T}}{2.0 \text{ s}} \right| = 4.17 \times 10^{-2} \text{ V} = 41.7 \text{ mV}$$

33.30. Model: Assume that the field is uniform in space over the coil. 

Visualize: We want an induced current so there must be an induced emf created by a changing flux.

Solve: To relate the emf and the current we need to know the resistance. From Equation 31.3,

$$R = \frac{\rho_{\text{Cu}}L}{A_{\text{wire}}} = \frac{\rho_{\text{Cu}}N2\pi r}{\pi r^2} = \frac{2(100)(1.7 \times 10^{-4} \text{ m} \Omega \text{m})(0.040 \text{ m})}{(2.5 \times 10^{-4} \text{ m})^2} = 2.18 \Omega$$

The magnetic field is perpendicular to the plane of the coil so the flux for a single loop of the coil is $\Phi = \vec{A} \cdot \vec{B} = BA$. if we take the normal to the coil to be in the same direction as the field. Using Faraday’s law,

$$\mathcal{E} = IR = NA \frac{d(BA)}{dt} = N \pi r^2 \frac{d(BA)}{dt} = \frac{IR}{N \pi r^2} = \frac{(2.0 \text{ A})(2.18 \Omega)}{100\pi(0.040 \text{ m})^2} = 8.67 \text{ T} / \text{s}$$
33.31. **Model:** Assume the wire is long enough so we can use the formula for the magnetic field of an "infinite" wire.

**Visualize:**

The magnetic field in the vicinity of the loop is due to the current in the wire and is perpendicular to the loop. The current is changing so the field and the flux through the loop are changing. This will create an induced emf and induced current in the loop.

**Solve:** The induced current depends on the induced emf and is

\[ I_{\text{loop}} = \frac{E_{\text{ind}}}{R} = \frac{1}{R} \frac{d\Phi}{dt} \]

The flux through a rectangular loop due to a wire was found in Example 33.5. The total flux is

\[ \Phi = \frac{\mu_0 I a}{2\pi} \ln \left( \frac{c + a}{c} \right) \]

\[ \Rightarrow I_{\text{loop}} = \frac{1}{R} \frac{\mu_0 b}{2\pi} \ln \left( \frac{c + a}{c} \right) \frac{dl}{dt} = \frac{4\pi \times 10^{-7} \text{ T m / A}}{0.010 \Omega} \left( 0.020 \text{ m} \right) \ln \left( \frac{0.030 \text{ m}}{0.010 \text{ m}} \right) (100 \text{ A / s}) = 43.9 \mu A \]

33.32. **Model:** Since the solenoid is fairly long compared to its diameter and the loop is located near the center, assume the solenoid field is uniform inside and zero outside.

**Visualize:** Please refer to Figure P33.32. The solenoid's magnetic field is perpendicular to the loop and creates a flux through the loop. This flux changes as the solenoid's current changes, causing an induced emf and corresponding induced current.

**Solve:** Using Faraday's law, the induced current is

\[ I_{\text{loop}} = \frac{E_{\text{ind}}}{R} = \frac{1}{R} \frac{d\Phi}{dt} = \frac{\mu_0 I a}{2\pi} \frac{dl}{dt} = \frac{\pi r_{\text{vol}}^2}{R} \frac{d}{dt} \left( \frac{\mu_0 N I}{l} \right) = \frac{\pi r_{\text{vol}}^2 \mu_0 N}{RL} \frac{dl}{dt} \]

We have used the fact that the field is approximately zero outside the solenoid, so the flux is confined to the area \( A_{\text{vol}} = \pi r_{\text{vol}}^2 \) of the solenoid, not the larger area of the loop itself. The current is constant from 0 s to 1 s and 2 s to 3 s, so \( dl/dt = 0 \text{ A/s} \) and \( I_{\text{loop}} = 0 \text{ A} \) during these intervals. The current is changing at the rate \( |dl/dt| = 40 \text{ A/s} \) during the interval 1 s to 2 s, so the current during this interval is

\[ I_{\text{loop}} = \frac{\pi (0.010 \text{ m})^2 (4\pi \times 10^{-7} \text{ T m / A}) (40 \text{ A / s})}{(0.10 \Omega)(0.010 \text{ m})} = 1.58 \times 10^{-4} \text{ A} = 158 \mu A \]

During the first half of this interval, from 1.0 s to 1.5 s, the field is to the right and decreasing. To oppose this decrease, the induced field must point to the right. This requires an induced current coming out of the page at the top of the loop—a positive current with the sign definition given in the problem. During the second half of this interval, from 1.5 s to 2.0 s, the field is to the right and increasing. To oppose this increase, the induced field must point to the right. This also requires a positive induced current coming out of the page at the top of the loop.

**Assess:** The induced current is proportional to the negative derivative of the solenoid current.
33.33. **Visualize:**

The bottom of the loop is on the x axis. The field is changing with time so the changing flux will produce an induced emf and corresponding induced current in the loop. The field strength varies in space as well as time.

**Solve:** Let's take the normal of the surface of the loop to be in the z direction so it is parallel to the magnetic field. The flux through the shaded strip is

$$d\Phi = \vec{B} \cdot d\vec{A} = B \, dA = B(b \, dy) = (0.80 \, y^2 \, t) \, b \, dy$$

To find the total flux we integrate over the area of the loop. We obtain

$$\Phi = \int_0^b (0.80 \, y^2 \, t) \, b \, dy = 0.80tb^4 \int_0^t y^2 \, dy = \frac{0.80tb^4}{3}$$

To find the induced current we take the time derivative of the flux:

$$I_{\text{loop}} = \frac{\mathcal{E}_{\text{loop}}}{R} = \frac{1}{R} \left( \frac{d}{dt} \left( \frac{0.80tb^4}{3} \right) \right) = \frac{0.80tb^4}{3R} = \frac{0.80(0.20 \, \text{m})^4}{3(0.50 \, \Omega)} = 8.53 \times 10^{-4} \, \text{A} = 0.853 \, \text{mA}$$

The induced current in the loop is a constant.

33.34. **Model:** Since the solenoid is fairly long compared to its diameter and the coil is located near the center, assume the solenoid field is uniform inside and zero outside.

**Visualize:** Please refer to Figure P33.34. The solenoid's magnetic field is parallel to the coil's axis and creates a flux through the coil. This flux changes as the solenoid's current changes, causing an induced emf and corresponding induced current in the coil.

**Solve:** The induced current from the induced emf is given by Faraday's law. We have

$$I_{\text{coil}} = \frac{\mathcal{E}_{\text{coil}}}{R} = \frac{1}{R} \, N_{\text{coil}} \left( \frac{d\Phi_{\text{coil}}}{dt} \right) = \frac{N_{\text{coil}} \pi r_{\text{coil}}^2}{R} \left( \frac{\mu_0 NI}{l} \right) = \frac{N_{\text{coil}} \pi r_{\text{coil}}^2 \mu_0 N}{RL} \left( \frac{dl}{dt} \right)$$

We used the fact that the coil flux is confined to the area \( A_{\text{coil}} = \pi r_{\text{coil}}^2 \) of the coil, not the larger area of the solenoid. The current is changing uniformly over the interval 0 s \( t \leq 0.02 \) s at the rate \( |dl/dt| = 50 \, \text{A/s} \) so the induced current during this interval is

$$I_{\text{coil}} = \frac{5\pi(0.0050 \, \text{m})^2(4\pi \times 10^{-7} \, \text{T m / A})(120)}{(0.1 \, \Omega)(0.080 \, \text{m})} = 3.7 \times 10^{-4} \, \text{A} = 0.37 \, \text{mA}$$

The current in the solenoid is changing steadily so the induced current in the coil is constant. The current is initially positive so the field is initially to the right and decreasing. The induced current will oppose this change and will therefore produce an induced field to the right. This requires an induced current in the coil coming out of the page at the top, so it is also positive. That is, it is clockwise when seen from the left.

$${I_{\text{coil}}(A)}$$

0.37 mA

$$t (\text{s})$$

0.02

33.35. **Model:** Assume that the magnetic field of coil 1 passes through coil 2 and that we can use the magnetic field of a solenoid for coil 1.

**Visualize:** Please refer to Figure P33.35. The field of coil 1 produces flux in coil 2. The changing current in coil 1 gives a changing flux in coil 2 and a corresponding induced emf and current in coil 2.
Solve: From 0 s to 0.1 s and 0.3 s to 0.4 s the current in coil 1 is constant so the current in coil 2 is zero. From 0.1 s to 0.3 s, the induced current from the induced emf is given by Faraday's law. The current in coil 2 is

\[ I_2 = \frac{E_2}{R} = \frac{1}{R} N_2 A_2 \frac{\dot{\Phi}}{dt} = \frac{1}{R} N_2 \pi r_2^2 \frac{d}{dt} \left( \frac{\mu_0 N_1 I_1}{l_2} \right) = \frac{N_2 \pi r_2^2 \mu_0 N_1}{R l_2} \frac{dI_1}{dt} \]

\[ = \frac{20 \pi (0.010 \text{ m})^2 (4 \pi \times 10^{-7} \text{ T m/A}) (20)}{(2 \Omega) (0.020 \text{ m})} = 79.5 \times 10^{-6} \text{ A} = 79 \mu \text{A} \]

We used the facts that the field of coil 1 is constant inside the loops of coil 2 and the flux is confined to the area \( A_2 = \pi r_2^2 \) of coil 2. Also, we used \( I_2 = N_2 = 20 \text{A} \) and \( dI_1/dt = 20 \text{ A/s} \). From 0.1 s to 0.2 s the current in coil 1 is initially negative so the field is initially to the right and the flux is decreasing. The induced current will oppose this change and will therefore produce a field to the right. This requires an induced current in coil 2 that comes out of the page at the top of the loops so it is negative. From 0.2 s to 0.3 s the current in coil 1 is positive so the field is to the left and the flux is increasing. The induced current will oppose this change and will therefore produce a field to the right. Again, this is a negative current.

\[ I_2 (\mu \text{A}) \]

\[ \begin{array}{c}
0 \\
0.2 \\
0.4 \\
-79
\end{array} \]

33.36. Model: Assume the magnetic field is uniform over the plane of the loop.

Visualize: The oscillating magnetic field strength produces a changing flux through the loop and an induced emf in the loop.

Solve: (a) The normal to the surface of the loop is in the same direction as the magnetic field so that \( \Phi = \mathbf{A} \cdot \mathbf{B} = BA \). The induced emf is

\[ E = \frac{d\Phi}{dt} = A \frac{dB}{dt} = \pi r^2 \frac{dB}{dt} = \pi r^2 \omega B_0 \cos \omega t \]

The cosine will oscillate between +1 and -1 so the maximum emf is

\[ E_{\text{max}} = \pi r^2 \omega B_0 = \pi^2 (2 \pi f) B_0 = 2 \pi^2 (0.125) (150 \times 10^6 \text{ Hz}) (20 \times 10^{-6} \text{ T}) = 0.925 \text{ V} \]

(b) If the loop is rotated so that the plane is perpendicular to the electric field, then the normal to the surface will be parallel to the magnetic field. There is no magnetic flux through the loop and no induced emf.

33.37. Model: Assume the field due to the solenoid is uniform inside and vanishes outside.

Visualize: The changing current in the solenoid produces a changing field and flux through the coil. This changing flux creates an induced emf in the coil.

Solve: The flux is only nonzero within the area of the solenoid, not the entire area of the coil. The flux is \( \Phi = \mathbf{A} \cdot \mathbf{B} = BA \) and the induced current in the coil is

\[ I_{\text{coil}} = \frac{E_{\text{coil}}}{R} = \frac{1}{R} N_{\text{coil}} \frac{d\Phi}{dt} = \frac{1}{R} N_{\text{coil}} A_{\text{coil}} \frac{d}{dt} \left( \frac{\mu_0 N_1 I_1}{l_1} \right) = \frac{N_{\text{coil}} \pi r_1^2 \mu_0 N_1}{R l_1} \frac{dI_1}{dt} \]

\[ = \frac{50 \pi (0.010 \text{ m})^2 (4 \pi \times 10^{-7} \text{ T m/A}) (200)}{(0.50 \Omega) (0.20 \text{ m})} = 50 \pi (0.010 \text{ m})^2 (4 \pi \times 10^{-7} \text{ T m/A}) (200) (0.50 \text{ A}) (2 \pi f) \cos (2 \pi f)
\]

Assess: The induced current oscillates because the current and field of the solenoid oscillate.
33.38. Model: Assume the field due to the solenoid is uniform inside and vanishes outside.

Visualize: The changing current in the solenoid produces a changing field and flux through the coil. This changing flux creates an induced emf in the coil.

Solve: The flux is only nonzero within the area of the solenoid, not the entire area of the coil. The induced current in the coil is

\[ I_{\text{coil}} = \frac{E_{\text{coil}}}{R} = \frac{1}{R} N_{\text{coil}} \frac{d\Phi}{dt} = \frac{1}{R} N_{\text{coil}} A \frac{dB}{dt} = \frac{1}{R} N_{\text{coil}} \pi r^2 \frac{d}{dt} \left( \mu_0 \pi I_s \right) \]

\[ = \frac{N_{\text{coil}} \pi r^2 \mu_0 N_s}{R} \frac{dI_s}{dt} = \frac{N_{\text{coil}} \pi r^2 \mu_0 N_s}{R} (2\pi f I_s) \cos(2\pi f t) \]

The maximum coil current occurs when the cosine is +1. The maximum current in the solenoid is

\[ I_0 = \frac{I_{\text{coil, max}} R L_i}{N_{\text{coil}} N_s \mu_0 (2\pi f)} = \frac{(0.20 \text{ A})(0.40 \Omega)(0.20 \text{ m})}{40\pi(0.015 \text{ m})^2 (4\pi \times 10^{-7} \text{ T m / A})(200)2\pi(60 \text{ Hz})} = 5.97 \text{ A} \]

Assess: This is a large current, but the induced current is large as well.


Visualize: An ideal transformer changes the voltage, but not the power (energy conservation).

Solve: (a) The primary and secondary voltages are related by Equation 33.23. We have

\[ V_2 = \frac{N_2}{N_1} V_1 \Rightarrow N_1 = \frac{V_1}{V_2} N_2 = 15,000 \frac{120}{100} = 12,500 \text{ turns} \]

(b) The input power equals the output power, and we recall that \( P = IAV \), so

\[ P_{\text{in}} = P_{\text{out}} \Rightarrow I_1AV_1 = I_2AV_2 \Rightarrow I_1 = \frac{I_2AV_2}{AV_1} = \frac{(250 \text{ A})120 \text{ V}}{15,000 \text{ V}} = 2.0 \text{ A} \]

Assess: These values seem reasonable, because houses have low voltage and high current while transmission lines have high voltage and low current.

33.40. Model: Assume the field changes abruptly at the boundary and is uniform.

Visualize: Please refer to Figure P33.40. As the loop enters the field region the amount of flux will change as more area has field penetrating it. This change in flux will create an induced emf and corresponding current. While the loop is moving at constant speed, the rate of change of the area is not constant because of the orientation of the loop. The loop is moving along the x-axis.

Solve: (a) If the edge of the loop enters the field region at \( t = 0 \) s, then the leading corner has moved a distance \( x = v_c t \) at time \( t \). The area of the loop with flux through it is

\[ A = 2(x) = x^2 = (v_c t)^2 \]

where we have used the fact that \( y = x \) since the sides of the loop are oriented at 45° to the horizontal. Take the surface normal of the loop to be into the page so that \( \Phi = A \cdot \vec{B} = BA \). The current in the loop is

\[ I = \frac{\varepsilon}{R} = \frac{1}{R} \frac{d\Phi}{dt} = \frac{1}{R} \frac{dB}{dt} \frac{d}{dt} (v_c t)^2 = \frac{1}{R} B (2v_c t) \left( \frac{2(0.80 \text{ T})(10 \text{ m / s})^2}{0.10 \Omega} \right) t = (1.6 \times 10^3 \text{ A}) t \]

The current is increasing at a constant rate. This expression is good until the loop is halfway into the field region. The time for the loop to be halfway is found as follows:

\[ \frac{10 \text{ cm}}{\sqrt{2}} = v_c t = (10 \text{ m / s}) t \Rightarrow t = 7.07 \times 10^{-3} \text{ s} = 7.07 \text{ ms} \]

At this time the current is 11.3 A. While the second half of the loop is moving into the field, the flux continues to increase, but at a slower rate. Therefore, the current will decrease at the same rate as it increased before, until the loop is completely in the field at \( t = 14.1 \text{ ms} \). After that the flux will not change and the current will be zero.

\[ I(t) \]

\[ \text{Graph} \]

\[ 11.3 \text{ A} \]

\[ 0 \text{ ms} \]

\[ 20 \text{ ms} \]
(b) The maximum current of 11.3 A occurs when the flux is changing the fastest and this occurs when the loop is halfway into the region of the field.

33.41. Model: The inner loop is small enough that the field due to the outer loop is essentially uniform over its area. Also, assume the current in the outer loop changes uniformly.
Visualize: The current in the outer loop creates a field at the location of the inner loop. Changing the current in the outer loop results in a change in flux and a corresponding induced emf and current in the inner loop.
Solve: The magnetic field due to the outer loop is perpendicular to the plane of the inner loop, so $\Phi_{\text{outer}} = \vec{A}_{\text{outer}} \cdot \vec{B}_{\text{outer}} = A_{\text{outer}} B_{\text{outer}}$, where $B_{\text{outer}} = \mu_0 I / 2 r_{\text{outer}}$ is the field of a current loop. The radius and angle don’t change with time, but $B_{\text{outer}}$ does. According to Faraday’s law, the induced emf is

$$I_{\text{outer}} = \frac{\Phi_{\text{outer}}}{R_{\text{outer}}} = \frac{1}{R_{\text{outer}}} \frac{\Phi}{dt} = \frac{\pi r_{\text{outer}}^2}{2 R_{\text{outer}} \mu_0} \frac{d}{dt} \left( \frac{\mu_0 I_{\text{outer}}}{2 r_{\text{outer}}} \right) = \frac{\pi r_{\text{outer}}^2 \mu_0}{2 R_{\text{outer}} \mu_0} \left| \frac{\Delta M_{\text{outer}}}{\Delta t} \right|$$

$$= \frac{\pi(0.0010 \text{ m})^2 (4\pi \times 10^{-7} \text{ T m} / \text{A})}{2(0.02 \text{ m})} \left| -1.0 \text{ T} - 1.0 \text{ T} \right| = 3.95 \times 10^{-8} \text{ A} = 39.5 \text{ nA}$$

33.42. Visualize: Please refer to Figure 33.27. The moving wire in a magnetic field results in a motional emf and a current in the loop.
Solve: (a) The induced emf is

$$E_{\text{loop}} = v l B = (100 \text{ m/s})(0.040 \text{ m})(1.0 \text{ T}) = 4.0 \text{ V}$$

(b) The total resistance of the square loop is $R = 4(0.010 \Omega) = 0.040 \Omega$. The induced current is

$$I = E_{\text{loop}} / R = (4.0 \text{ V})/(0.040 \Omega) = 100 \text{ A}$$

(c) The slide wire generates the emf, $E_{\text{loop}}$. However, the slide wire also has an “internal resistance” due to the resistance of the slide wire itself—namely $r = 0.01 \Omega$. Thus, the slide wire acts like a battery with an internal resistance. The potential difference between the ends is

$$\Delta V = E_{\text{loop}} - Ir = 4.0 \text{ V} - (100 \text{ A})(0.01 \Omega) = 3.0 \text{ V}$$

It is this 3.0 V that drives the 100 A current through the remaining 0.03 Ω resistance of the “external circuit.”
Assess: The slide wire system acts as an emf just like a battery does.

33.43. Model: Assume that the field is uniform in space over the loop.
Visualize:

```
+-------------------+   +-------------------+
|                  |   |                  |
|   0Ω             |   |   0Ω             |
|                  |   |                  |
|   0Ω             |   |   0Ω             |
|                  |   |                  |
+-------------------+   +-------------------+
   1.0 Ω          
```

The moving slide wire in a magnetic field develops a motional emf and a corresponding current in the wire and the rails. The current through the resistor will cause energy dissipation and subsequent warming.
Solve: (a) The induced emf depends on the changing flux, not on where the resistance in the loop is located. We have

$$E_{\text{loop}} = v l B = (10 \text{ m/s})(0.20 \text{ m})(0.10 \text{ T}) = 0.20 \text{ V}$$

The total resistance around the loop is due entirely to the carbon resistor, so the induced current is

$$I = E_{\text{loop}} / R = (0.20 \text{ V})/(1.0 \Omega) = 0.20 \text{ A}$$

(b) To move with a constant velocity the acceleration must be zero, so the pulling force must balance the retarding magnetic force on the induced current in the slide wire. Thus,

$$F_{\text{pull}} = F_{\text{mag}} = ilB = (0.20 \text{ A})(0.20 \text{ m})(0.1 \text{ T}) = 4.0 \times 10^{-3} \text{ N}$$

(c) The current in the resistor will result in power being dissipated. The power is

$$P = IR = (0.20 \text{ A})^2 (1 \Omega) = 0.040 \text{ W} = 0.040 \text{ J/s}$$
During a 10 second period $Q = 0.40 \, \text{J}$ of energy is dissipated by the current, increasing the internal energy of the carbon resistor and raising its temperature. This is, effectively, a heat source. The heat is related to the temperature rise by $Q = mc\Delta T$, where $c$ is the specific heat of carbon. Thus,

$$\Delta T = \frac{Q}{mc} = \frac{0.40 \, \text{J}}{(5.0 \times 10^{-3} \, \text{kg})(710 \, \text{J/kg \degree C})} = 11 \degree \text{C}$$

**Assess:** This is a significant change in the temperature of the resistor.

### 33.44. Model:
Assume there is no resistance in the rails. If there is any resistance, it is accounted for by the resistor.

**Visualize:** Please refer to Figure P33.44. The moving wire will have a motional emf that produces a current in the loop.

**Solve:**
(a) At constant velocity the external pushing force is balanced by the magnetic force, so

$$F_{\text{push}} = F_{\text{mag}} = ILB = \left(\frac{Blv}{R}\right)IL = \frac{(0.50 \, \text{T})(0.10 \, \text{m})(0.50 \, \text{m/s})}{2.0 \, \Omega} = 0.625 \times 10^{-4} \, \text{N}$$

(b) The power is

$$P = Fv = (6.25 \times 10^{-4} \, \text{N})(0.50 \, \text{m/s}) = 3.13 \times 10^{-4} \, \text{W}$$

(c) The flux is out of the page and decreasing and the induced current-field will oppose the change. The induced field must have a flux that is out of the page so the current will be counterclockwise.

The magnitude of the current is

$$I = \left(\frac{Blv}{R}\right) = \frac{(0.50 \, \text{T})(0.10 \, \text{m})(0.50 \, \text{m/s})}{2.0 \, \Omega} = 1.25 \times 10^{-2} \, \text{A}$$

(d) The power is

$$P = I^2R = \left(1.25 \times 10^{-2} \, \text{A}\right)^2 (2.0 \, \Omega) = 3.13 \times 10^{-4} \, \text{W}$$

**Assess:** From energy conservation we see that the mechanical energy put in by the pushing force shows up as electrical energy in the resistor.

### 33.45. Model:
Assume that the magnetic field is uniform in the region of the loop.

**Visualize:** Please refer to Figure P33.45. The rotating semicircle will change the area of the loop and therefore the flux through the loop. This changing flux will produce an induced emf and corresponding current in the bulb.

**Solve:**
(a) The spinning semicircle has a normal to the surface that changes in time, so while the magnetic field is constant, the area is changing. The flux through in the lower portion of the circuit does not change and will not contribute to the emf. Only the flux in the part of the loop containing the rotating semicircle will change. The flux associated with the semicircle is

$$\Phi = \vec{A} \cdot \vec{B} = BA = BACos\theta = BAcos(2\pi f)$$

where $\theta = 2\pi f$ is the angle between the normal of the rotating semicircle and the magnetic field and $A$ is the area of the semicircle. The induced current from the induced emf is given by Faraday's law. We have

$$I = \frac{\mathcal{E}}{R} = \left| \frac{d\Phi}{dt} \right| = \frac{1}{R} \left| \frac{d}{dt} (BAcos(2\pi f)) \right| = \frac{B \pi^2}{R} \frac{n^2}{2} 2\pi \sin(2\pi f)$$

$$= \frac{2(0.20 \, \text{T})\pi^2(0.050 \, \text{m})^2}{2(1.0 \, \Omega)} \times f \sin(2\pi f) = 4.93 \times 10^{-3} f \sin(2\pi f) \, \text{A}$$

where the frequency $f$ is in Hz.

(b) We can now solve for the frequency necessary to achieve a certain current. From our study of DC circuits we know how power relates to resistance:

$$P = I^2R \Rightarrow I = \sqrt{P/R} = \sqrt{4.0 \, \text{W}/1.0 \Omega} = 2.0 \, \text{A}$$

The maximum of the sine function is +1, so the maximum current is

$$I_{\text{max}} = 4.93 \times 10^{-3} f \, \text{A} \Rightarrow f = \frac{2.0 \, \text{A}}{4.93 \times 10^{-7} \, \text{A} \, \text{s}} = 405 \, \text{Hz}$$

**Assess:** This is not a reasonable frequency to obtain by hand.
33.46. **Model:** Assume the magnetic field is uniform over region where the bar is sliding and that friction between the bar and the rails is zero. **Visualize:** Please refer to Figure P33.46. The battery will produce a current in the rails and bar and the bar will experience a force. With the battery connected as shown in the figure, the current in the bar will be down and by the right-hand rule the force on the bar will be to the right. The motion of the bar will change the flux through the loop and there will be an induced emf that opposes the change. **Solve:** (a) As the bar speeds up the induced emf will get larger until finally it equals the battery emf. At that point, the current will go to zero and the bar will continue to move at a constant velocity. We have

$$E = Blv = E_{\text{bat}} \Rightarrow v_{\text{term}} = \frac{E_{\text{bat}}}{Bl}$$

(b) The terminal velocity is

$$v_{\text{term}} = \frac{1.0 \text{ V}}{(0.50 \text{ T})(0.060 \text{ m})} = 33.3 \text{ m/s}$$

**Assess:** This is pretty fast, about 70 mph.

33.47. **Model:** Assume the magnetic field is uniform in the region of the loop. **Visualize:**

The moving wire creates a changing area and corresponding change in flux. This produces an induced emf and induced current. The flux through the loop depends on the size and orientation of the loop. **Solve:** (a) The normal to the surface is perpendicular to the loop and the flux is $\Phi_{\text{loop}} = \vec{A} \cdot \vec{B} = AB \cos \theta$. We can get the current from Faraday’s law. Since the loop area is $A = l x$, We have

$$I = \frac{\mathcal{E}}{R} = \frac{1}{R} \left[ \frac{d\Phi}{dt} \right] = \frac{1}{R} \frac{d}{dt} (l x B \cos \theta) = \frac{Bl \cos \theta}{R} \frac{dx}{dt} = \frac{Blv \cos \theta}{R}$$

(b) Using the free-body diagram shown in the figure, we can apply Newton’s second law. The magnetic force on a straight, current-carrying wire is $F_m = II B$ and is horizontal. Using the current $I$ from part (a) gives

$$\sum F_i = -F_m \cos \theta + mg \sin \theta = -\frac{B^2 l^2 v \cos^2 \theta}{R} + mg \sin \theta = ma.$$  

Terminal velocity is reached when $a$, drops to zero. In this case, the two terms are equal and we have

$$v_{\text{term}} = \frac{mg \tan \theta}{l^2 B^2 \cos \theta}$$
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33.48. Model: Assume the magnetic field is uniform and that the change in field occurs abruptly.
Visualize:

The figure shows the front of the loop, with the field coming out of the page. As the loop falls, the flux will decrease. This will induce a current to oppose the decrease, so the induced current will flow counterclockwise. As this current passes through the top of loop, it experiences an upward magnetic force. There is no force on the bottom of the loop because $B = 0 \, \text{T}$ at the bottom. So there is an upward force—a retarding force—on the loop as it falls in the field. As the loop speeds up the retarding force will become larger until it balances the weight.

Solve: (a) The force on the current in the top arm of the loop is $F_{\text{mag}} = ilB$. The forces on the side arms of the loop cancel. The induced current is

$$I = \frac{\mathcal{E}}{R} = \frac{1}{R} \left| \frac{d\Phi}{dt} \right| = \frac{1}{R} \left| \frac{d}{dt} AB \right| = \frac{B}{R} \left| \frac{dx}{dt} \right| = \frac{Blv}{R}$$

Consequently, the retarding magnetic force is

$$F_{\text{m}} = \frac{i^2 B^2 v}{R} = \left( \frac{i^2 B^2}{R} \right) v$$

The important point is that $F_{\text{m}}$ is proportional to the speed $v$. As the loop begins to fall and its speed increases, so does the retarding force. Within a very short time, $F_{\text{m}}$ will increase in size to where it matches the weight $w = mg$. At that point, there is no net force on the loop, so it will continue to fall at a constant speed. The condition that the magnetic force equals the weight is

$$\left( \frac{i^2 B^2}{R} \right) v = mg \Rightarrow v = \frac{mgR}{i^2 B^2} = \frac{(0.010 \, \text{kg})(9.80 \, \text{m/s}^2)(0.010 \Omega)}{(0.20 \, \text{m})^2(1.0 \, \text{T})^2} = 0.0245 \, \text{m/s}$$

(b) The loop has to fall 10 cm for the top edge to leave the field. The time it takes to move this distance is

$$\Delta t = \frac{10 \, \text{cm}}{0.10 \, \text{m}/0.0245 \, \text{m/s}} = 4.08 \, \text{s}$$

If there were no field, the loop would drop in free fall. In this case, the time can be obtained using kinematics and the particle model for the loop. We have

$$(y_f - y_i) = v_i \Delta t + (1/2) a (\Delta t)^2 \Rightarrow -0.10 \, \text{m} = 0 \, \text{m} + (1/2)(-9.8 \, \text{m/s}^2)(\Delta t)^2 \Rightarrow \Delta t = 0.14 \, \text{s}$$

Assess: It takes the loop much longer to fall in the field. This is an example of magnetic braking.

33.49. Model: Assume the magnetic field is uniform over the loop.
Visualize: Please refer to Figure 33.49. As the wire falls, the flux into the page will increase. This will induce a current to oppose the increase, so the induced current will flow counterclockwise. As this current passes through the slide wire, it experiences an upward magnetic force. So there is an upward force—a retarding force—on the wire as it falls in the field. As the wire speeds up the retarding force will become larger until it balances the weight.

Solve: (a) The force on the current-carrying slide wire is $F_{\text{mag}} = ilB$. The induced current is

$$I = \frac{\mathcal{E}}{R} = \frac{1}{R} \left| \frac{d\Phi}{dt} \right| = \frac{1}{R} \left| \frac{d}{dt} AB \right| = \frac{B}{R} \left| \frac{dx}{dt} \right| = \frac{Blv}{R}$$

Consequently, the retarding magnetic force is

$$F_{\text{m}} = \frac{i^2 B^2 v}{R} = \left( \frac{i^2 B^2}{R} \right) v$$
The important point is that $F_{\text{emi}}$ is proportional to the speed $v$. As the wire begins to fall and its speed increases, so does the retarding force. Within a very short time, $F_{\text{emi}}$ will increase in size to where it matches the weight $w = mg$. At that point, there is no net force on the loop, so it will continue to fall at a constant speed. The condition that the magnetic force equals the weight is

$$
\left( \frac{I^2 B^2}{R} \right) v_{\text{emi}} = mg \Rightarrow v_{\text{emi}} = \frac{mgR}{I^2 B^2}
$$

(b) The terminal speed is

$$
v_{\text{emi}} = \frac{(0.010 \text{ kg})(9.80 \text{ m/s}^2)(0.10 \Omega)}{(0.20 \text{ m})^2(0.50 \text{ T})} = 0.98 \text{ m/s}
$$

33.50. **Model:** Assume the coil is moved to a location where the magnetic field is zero.

**Visualize:**

![Diagram of a coil and magnetic field](image)

The removal of the coil from the field will change the flux and produce an induced emf and corresponding induced current. The current will charge the capacitor.

**Solve:** The induced current is

$$
I_{\text{coli}} = \frac{\mathcal{E}_{\text{coli}}}{R} = \frac{N |\Delta \Phi|}{R |dt|}
$$

The definition of current is $I = dq/dt$. Consequently, the charge flow through the coil and onto the capacitor is given by

$$
\frac{dq}{dt} = \frac{N |\Delta \Phi|}{R \Delta t} \Rightarrow \Delta q = \frac{N |\Delta \Phi|}{R} \Delta t \Rightarrow \Delta q = \frac{N}{R} |\Delta \Phi| = \frac{N}{R} (\Phi_i - \Phi_f)
$$

We are only interested in the total charge that flows due to the change in flux and not the details of the time dependence. In this case, the flux is changed by physically pulling the coil out of the field. Since the coil is oriented for maximum flux, the initial flux through the coil is

$$
\Phi_i = \pi r^2 B = \pi (0.0050 \text{ m})^2 (0.0010 \text{ T}) = 7.85 \times 10^{-8} \text{ Wb}
$$

After being pulled from the field, the final flux is $\Phi_f = 0 \text{ Wb}$. The charge that flows onto the capacitor is

$$
\Delta q = \frac{(10)0 \text{ Wb} - 7.85 \times 10^{-8} \text{ Wb}}{0.20 \Omega} = 3.93 \times 10^{-9} \text{ C} \Rightarrow \Delta V_c = \frac{\Delta q}{C} = \frac{3.93 \times 10^{-9} \text{ C}}{1.0 \times 10^{-6} \text{ F}} = 3.93 \text{ V}
$$

33.51. **Model:** Assume the field is uniform in the region of the coil.

**Visualize:**

![Diagram of a coil and magnetic field](image)

The rotation of the coil in the field will change the flux and produce an induced emf and a corresponding induced current. The current will charge the capacitor.

**Solve:** The induced current is

$$
I_{\text{coli}} = \frac{\mathcal{E}_{\text{coli}}}{R} = \frac{N |\Delta \Phi|}{R |dt|}
$$

The definition of current is $I = dq/dt$. Consequently, the charge flow through the coil and onto the capacitor is given by

$$
\frac{dq}{dt} = \frac{N |\Delta \Phi|}{R \Delta t} \Rightarrow \Delta q = \frac{N |\Delta \Phi|}{R} \Delta t \Rightarrow \Delta q = \frac{N}{R} |\Delta \Phi| = \frac{N}{R} (\Phi_i - \Phi_f)
$$
We are only interested in the total charge that flows due to the change in flux and not the details of the time dependence. In this case, the flux is changed by physically rotating the coil in the field. The flux is \( \Phi = A \cdot B = AB \cos \theta \). The change in flux is
\[
\Delta \Phi = AB(\cos \theta_f - \cos \theta_i) = \pi(0.020 \text{ m})^2(55 \times 10^{-6} \text{ T})(\cos 30^\circ - \cos 210^\circ) = 1.2 \times 10^{-7} \text{ Wb}
\]
Note that the field is 60° from the horizontal and the normal to the plane of the loop is vertical. The final angle, when \( \theta_f \) points down, is \( \theta_f = 30^\circ \), so the initial angle is \( \theta_i = \theta_i + 180^\circ = 210^\circ \). The charge that flows onto the capacitor is
\[
\Delta q = \frac{N}{R}(\Phi_f - \Phi_i) = \frac{200(1.2 \times 10^{-7} \text{ Wb})}{2.0 \Omega} = 1.2 \times 10^{-5} \text{ C} \Rightarrow \Delta V_c = \frac{\Delta q}{C} = \frac{1.2 \times 10^{-5} \text{ C}}{1.0 \times 10^{-6} \text{ F}} = 12 \text{ V}
\]

33.52. **Visualize:** The battery will cause a current through the inductor. Hence, there will be stored energy. **Solve:** The energy stored in the inductor depends on the inductance and the current. According to Ohm's law,
\[
I = \frac{\Delta V}{R} = \frac{12 \text{ V}}{4.0 \Omega + 2.0 \Omega} = 2.0 \text{ A} \quad U_L = \frac{1}{2}LI^2 = \frac{1}{2}(100 \times 10^{-3} \text{ H})(2.0 \text{ A})^2 = 0.20 \text{ J}
\]

33.53. **Visualize:** The current through the inductor has energy stored in the magnetic field. **Solve:** The energy stored in the inductor depends on the inductance and the current. We know the energy and the current so we can get the inductance. We have
\[
U_L = \frac{1}{2}LI^2 \Rightarrow L = \frac{2U_L}{I^2} = \frac{2(1.0 \times 10^{-5} \text{ J})}{(0.20 \text{ A})^2} = 5.0 \times 10^{-2} \text{ H}
\]
We know the flux per turn for a given current so we can relate the inductance to the magnetic field and the flux per turn. The flux through one turn of the solenoid is
\[
\Phi^{(1)} = B_{in}A = \left(\frac{\mu_0 NI}{l}\right)A = \left(\frac{\mu_0 NA}{l}\right)l = \left(\frac{\mu_0 NA}{l}\right)
\]
But the inductance is
\[
L_{tot} = \frac{\mu_0 N^2 A}{l} = \left(\frac{\mu_0 NA}{l}\right)N \Rightarrow \left(\frac{\mu_0 NA}{l}\right) = \frac{L_{tot}}{N}
\]
Substituting into the previous expression,
\[
\Phi^{(1)} = \left(\frac{L_{tot}}{N}\right)l \Rightarrow N = \frac{L_{tot}}{\Phi^{(1)}}I = \frac{5.0 \times 10^{-2} \text{ H}(0.200 \text{ A})}{20 \times 10^{-6} \text{ Wb/turn}} = 500 \text{ turns}
\]

33.54. **Visualize:** The changing current produces an emf across the inductor. **Solve:** When the current is changing we know the induced emf, so we can get the inductance. We have
\[
\Delta V_L = -L \frac{dl}{dt} \Rightarrow L = \frac{\Delta V}{dl/dt} = \frac{0.20 \text{ V}}{10.0 \text{ A/s}} = 20 \text{ mH}
\]
We know the flux per turn for a given current so we can connect the inductance to the magnetic field and the flux per turn. The flux through one turn of the solenoid is
\[
\Phi^{(1)} = B_{in}A = \left(\frac{\mu_0 NI}{l}\right)A = \left(\frac{\mu_0 NA}{l}\right)l
\]
But the inductance is
\[
L_{tot} = \frac{\mu_0 N^2 A}{l} = \left(\frac{\mu_0 NA}{l}\right)N \Rightarrow \left(\frac{\mu_0 NA}{l}\right) = \frac{L_{tot}}{N}
\]
Substituting into the previous expression we get
\[
\Phi^{(1)} = \left(\frac{L_{tot}}{N}\right)I \Rightarrow N = \frac{L_{tot}}{\Phi^{(1)}}I = \frac{(20 \times 10^{-3} \text{ H})(0.10 \text{ A})}{5.0 \times 10^{-6} \text{ Wb/turn}} = 400 \text{ turns}
\]
33.55. **Model:** Assume the wire is "infinite" and the solenoid is long enough so that the field is constant.  
**Visualize:** The energy density depends on the field strength in a particular region of space.  
**Solve:** (a) Using the formula for the magnetic field of a long wire,  
\[
B = \frac{\mu_0 I}{2\pi r_{\text{wire}}} = \frac{(4\pi \times 10^{-7} \text{ T m/A})(1.0 \text{ A})}{2\pi(0.00050 \text{ m})} = 4.0 \times 10^{-4} \text{ T}
\]
\[
u_B = \frac{1}{2\mu_0} B^2 = \frac{(4.0 \times 10^{-4} \text{ T})^2}{2(4\pi \times 10^{-7} \text{ T m/A})} = 6.37 \times 10^{-2} \text{ J/m}^3
\]
(b) The number of turns and length of the solenoid are not known. However, the length can be related to the diameter of the winding wire with the equation \( l = N d_{\text{wire}} \). We have
\[
B = \frac{\mu_0 N I}{l} = \frac{\mu_0 N I}{N d_{\text{wire}}} = \frac{(4\pi \times 10^{-7} \text{ T m/A})(1.0 \text{ A})}{0.0010 \text{ m}} = 1.26 \times 10^{-3} \text{ T}
\]
\[
u_B = \frac{1}{2\mu_0} B^2 = \frac{(1.26 \times 10^{-3} \text{ T})^2}{2(4\pi \times 10^{-7} \text{ T m/A})} = 0.628 \text{ J/m}^3
\]
**Assess:** Note that the energy density is larger in the solenoid than outside the wire for the same current.

33.56. **Model:** Assume the straight wire in part (b) is "infinite."  
**Visualize:** The energy density depends on the field strength in a particular region of space.  
**Solve:** (a) We will use the formula for the field at the center of a current loop and then find the energy density. We have
\[
B = \frac{\mu_0 I}{2\pi R_{\text{loop}}} = \frac{(4\pi \times 10^{-7} \text{ T m/A})(1.0 \text{ A})}{2(0.020 \text{ m})} = 3.14 \times 10^{-5} \text{ T}
\]
\[
u_B = \frac{1}{2\mu_0} B^2 = \frac{(3.14 \times 10^{-5} \text{ T})^2}{2(4\pi \times 10^{-7} \text{ T m/A})} = 3.93 \times 10^{-7} \text{ J/m}^3
\]
(b) We will use the standard formula for the magnetic field of a long straight wire. We have
\[
B = \frac{\mu_0 I}{2\pi r} \Rightarrow \nu_B = \frac{1}{2\mu_0} B^2 = \frac{1}{2\mu_0} \left( \frac{\mu_0 I}{2\pi r} \right)^2 \Rightarrow I = \left( \frac{8\pi^2 \nu_B}{\mu_0} \right)^{1/2} \left( \frac{8\pi^2 (0.020 \text{ m})^2 (3.93 \times 10^{-7} \text{ J/m}^3)}{(4\pi \times 10^{-7} \text{ T m/A})} \right)^{1/2} = 3.14 \text{ A}
\]

33.57. **Model:** Assume the magnetic field is constant throughout the atmosphere.  
**Visualize:** The energy density depends on the field strength in a particular region of space. The total energy is the energy density times the volume.  
**Solve:** (a) The atmosphere is a spherical shell between the surface of the earth at \( R_e \) and the top of the atmosphere at \( R_e + h \) where \( h \) is the thickness of the atmosphere. We have
\[
\nu_B = \frac{1}{2\mu_0} B^2 = \frac{(5.0 \times 10^{-6} \text{ T})^2}{2(4\pi \times 10^{-7} \text{ T m/A})} = 9.95 \times 10^{-5} \text{ J/m}^3
\]
\[
V_{\text{atm}} = \frac{4}{3}\pi \left( (R_e + h)^3 - R_e^3 \right) = \frac{4}{3}\pi \left[ (6.37 \times 10^6 \text{ m} + 20 \times 10^6 \text{ m})^3 - (6.37 \times 10^6 \text{ m})^3 \right] = 1.02 \times 10^{19} \text{ m}^3
\]
\[
\Rightarrow U_{\text{atm}} = \nu_B V_{\text{atm}} = (9.95 \times 10^{-5} \text{ J/m}^3)(1.02 \times 10^{19} \text{ m}^3) = 1.0 \times 10^{14} \text{ J}
\]
(b) The ratio is
\[
\frac{U_{\text{atm}}}{U_{\text{tot}}} = \frac{1.0 \times 10^{14} \text{ J}}{4.0 \times 10^{19} \text{ J}} = 0.25 \times 10^{-5} = 0.25 \%
\]
**Assess:** Not enough to do much good.

33.58. **Model:** Assume the solenoid is long enough that we can approximate the field as being constant inside and zero outside.  
**Visualize:** The energy density depends on the field strength in a particular region of space.
Solve: (a) We can use the given field strength to find the energy density and then determine the volume of a cylinder. We have

\[ u_n = \frac{1}{2\mu_0} B^2 = \frac{(5.0 \text{ T})^2}{2(4\pi \times 10^{-7} \text{ T m / A})} = 9.95 \times 10^6 \text{ J / m}^3 \]

\( V_{\text{sol}} = \pi r^2 l = \pi (0.20 \text{ m})^2 (1.0 \text{ m}) = 0.126 \text{ m}^3 \Rightarrow U_{\text{sol}} = u_n V_{\text{sol}} = (9.95 \times 10^6 \text{ J / m}^3)(0.126 \text{ m}^3) = 1.25 \times 10^6 \text{ J} \)

(b) Using the magnetic field of the solenoid, the number of turns is calculated as follows:

\[ B = \frac{\mu_0 N I}{l} \Rightarrow N = \frac{IB}{\mu_0 l} = \frac{(1.0 \text{ m})(5.0 \text{ T})}{(4\pi \times 10^{-7} \text{ T m / A})(100 \text{ A})} = 4.0 \times 10^4 \]

Assess: This is a reasonable number for a solenoid that is a meter long.

33.59. Model: Assume the field is constant over the cross section of the patient.
Visualize: The changing field results in a changing flux and an induced emf.
Solve: To determine the largest emf, assume the normal to the surface is parallel to the field. From Faraday’s law, the emf is

\[ \mathcal{E} = \left| \frac{d\Phi}{dt} \right| = A \left| \frac{dB}{dt} \right| = A \left| \frac{\Delta B}{\Delta t} \right| \]

The time interval acceptable for the change is

\[ \Delta t = \frac{A \left| \frac{\Delta B}{\Delta t} \right|}{\mathcal{E}} = \frac{(0.060 \text{ m}^2)(5.0 \text{ T} - 0 \text{ T})}{0.10 \text{ V}} = 3.0 \text{ s} \]

Assess: This is a reasonable amount of time in which to change the field.

33.60. Model: Assume the magnetic field is uniform over the loop.
Visualize: The motion of the eye will change the orientation of the loop relative to the fixed field direction resulting in an induced emf.
Solve: We are just interested in the emf due to the motion of the eye so we can ignore the details of the time dependence of the change. From Faraday’s law,

\[ \mathcal{E}_{\text{coil}} = N \left| \frac{d\Phi}{dt} \right| = N \left| \frac{\Delta \Phi}{\Delta t} \right| = N A B \left| \frac{\Delta \cos \theta}{\Delta t} \right| = \frac{N A B (\cos \theta_1 - \cos \theta_2)}{\Delta t} = \frac{20\pi (3 \times 10^{-3} \text{ m})^2 (1.0 \text{ T}) (\cos 85^\circ - \cos 90^\circ)}{0.20 \text{ s}} = 2.46 \times 10^{-4} \text{ V} \]

Assess: This is a reasonable emf to measure, although you might need some amplification.

33.61. Model: Assume we can ignore the sharp corners when the current changes abruptly.
Visualize: The changing current produces a changing flux, an induced emf, and a corresponding potential difference.
Solve: Break the current into time intervals over which the current is changing linearly or not at all. For the intervals 1 ms to 2 ms and 4 ms to 5 ms, the current does not change. For the interval 0 s to 1 ms, the current goes from 0 A to 1 A, so the potential difference is zero. For the interval 5 ms to 6 ms, the potential difference is

\[ \Delta V_L = -L \frac{\Delta I}{\Delta t} = -L \frac{1 A - 0 A}{(1 \text{ s} - 0 \text{ s}) \times 10^{-3}} = -10 \text{ V} \]

Similarly for the interval 2 ms to 4 ms, the potential difference is +5 V. For the interval 5 ms to 6 ms, the potential difference is +20 V.

\[ \Delta V_L (V) \]

Assess: The potential difference is proportional to the negative slope of the current versus time graph.
33.62. **Model:** Assume we can ignore the sharp corners when the current changes abruptly.  
**Visualize:** The changing current produces a changing flux, an induced emf, and a corresponding potential difference.  
**Solve:** Break the current into time intervals over which the current is changing linearly or not at all. For the intervals 2 ms to 3 ms and 5 ms to 6 ms, the current does not change, so the potential difference is zero. For the interval 0 ms to 2 ms, the current goes from 1 A to -1 A, so the potential difference is  
\[ \Delta V_L = -L \frac{dI}{dt} = -L \frac{\Delta I}{\Delta t} \Rightarrow \Delta V_L = \left(10 \times 10^{-3} \text{ H}\right) \frac{-1 \text{ A} - 1 \text{ A}}{(2 \text{ s} - 0 \text{ s}) 	imes 10^{-3}} = 10 \text{ V} \]
Similarly for the interval 3 ms to 4 ms, the potential difference is -10 V. For the interval 4 ms to 5 ms, the potential difference is +20 V.

```
\[ \Delta V_L (V) \]
```

```
\[ \begin{array}{c|c}
 t (ms) & 0 & 1 & 3 & 4 & 5 & 6 \\
\hline
\Delta V_L (V) & 0 & 10 & -10 & 0 & +20 & \\
\end{array} \]
```

**Assess:** The potential difference is proportional to the negative slope of the current versus time graph.

33.63. **Model:** Assume we can ignore the edges when the potential difference changes abruptly.  
**Visualize:** The existence of a potential difference indicates there is a changing current. If the potential difference is zero, the current is constant.  
**Solve:** Break the potential difference into time intervals and determine the corresponding rate of change of the current for each time interval. Knowing the starting current for an interval, we can determine the current at the end of the interval by finding the change. For the intervals 10 ms to 20 ms and 30 ms to 40 ms, the potential difference is zero, so the current remains the same as at the start of the interval. For the interval 0 ms to 10 ms, the potential difference is 1 V. The rate of change of the current is calculated as follows:  
\[ \Delta V_L = -L \frac{dI}{dt} = -L \frac{\Delta I}{\Delta t} \Rightarrow \Delta I_{0 \rightarrow 10} = \left(1 \text{ V}\right) \frac{(1 \text{ V})(10 \text{ s} - 0 \text{ s}) \times 10^{-3} \text{ s}}{50 \times 10^{-3} \text{ H}} = -0.20 \text{ A} \]
To find the current at 10 ms we need the current at the start of the interval. We have  
\[ I(10 \text{ ms}) = I(0 \text{ ms}) + \Delta I_{0 \rightarrow 10} = 0.20 \text{ A} + (-0.20 \text{ A}) = 0 \text{ A} \]
A constant potential difference implies that the current is changing linearly. So, the current starts at 0.2 A and goes linearly to 0 A over the first 10 ms. For the interval 10 ms to 20 ms, the current remains at 0 A. For the interval 20 ms to 30 ms, the current goes linearly from 0 A to -0.4 A. For the interval 30 ms to 40 ms, the current remains at -0.4 A.

```
\[ I (A) \]
```

```
\[ \begin{array}{c|c}
 t (ms) & 0 & 10 & 20 & 30 & 40 \\
\hline
I (A) & 0 & 0.20 & 0 & -0.4 & -0.4 \\
\end{array} \]
```

33.64. **Model:** Assume we can ignore the edges when the potential difference changes abruptly.  
**Visualize:** The existence of a potential difference indicates there is a changing current. If the potential difference is zero, the current is constant.  
**Solve:** Break the potential difference into time intervals and determine the corresponding rate of change of the current for each time interval. Knowing the starting current for an interval, we can determine the current at the end of the interval by finding the change. For the intervals 0 ms to 10 ms and 30 ms to 40 ms, the potential difference is...
zero, so the current remains the same as at the start of the interval. For the interval 10 ms to 20 ms, the potential difference is 1 V. The rate of change of the current is calculated as follows:

$$\Delta V_L = -L \frac{dl}{dt} = -L \frac{\Delta I}{\Delta t} = \frac{-\Delta V_L \Delta t}{L} = \frac{-(1 \text{ V})(20 \text{ s} - 10 \text{ s}) \times 10^{-3}}{100 \times 10^{-3} \text{ H}} = -0.10 \text{ A}$$

To find the current at 20 ms we need the current at the start of the interval. We have

$$I(20) = I(10) + \Delta I_{10-20} = 0.10 \text{ A} + (-0.10 \text{ A}) = 0 \text{ A}$$

A constant potential difference implies that the current is changing linearly. So, the current starts at 0.10 A and goes linearly to 0 A during the 10 ms to 20 ms interval. Similarly, from 0 ms to 10 ms, the current remains at its starting value of 0.10 A. For the interval 20 ms to 30 ms, the current goes linearly from 0 A to 0.10 A. For the interval 30 ms to 40 ms, the current remains at 0.10 A.

33.65. Visualize: The current through the inductor is changing with time which leads to a changing potential difference across the inductor.

Solve: (a) To find the potential difference we differentiate the current. We have

$$\Delta V_L = -L \frac{dl}{dt} = -L \frac{d}{dt} \left(I_0 e^{-st}\right) = \left(\frac{L I_0}{r}\right) e^{-st}$$

(b) For $t = 1$ ms, the potential difference is

$$\Delta V_L = \left(\frac{20 \times 10^{-3} \text{ H}}{1.0 \times 10^{-3} \text{ s}}\right) \left[50 \times 10^{-3} \text{ A}\right] e^{-1.0 \times 10^{-3} \times 1.0 \times 10^{-3} s} = (1.0 \text{ V}) e^{-1} = 0.37 \text{ V}$$

Similar calculations give 1.0 V, 0.13 V, and 0.05 V for $t = 0$ ms, 2 ms, and 3 ms.

(c) $\Delta V_L$ ($\text{V}$)

33.66. Visualize: The potential difference across the inductor depends on the rate of change of the current with respect to time.

Solve: (a) We can find the potential difference by taking a derivative. We have

$$\Delta V_L = -L \frac{dl}{dt} = -L \frac{d}{dt} I_0 \sin \omega t = -L \omega I_0 \cos \omega t$$
(b) The cosine function oscillates between +1 and −1 so we ignore it and the negative sign. The maximum potential difference is

\[ \Delta V_{\text{L, max}} = \omega L_0 \Rightarrow I_0 = \frac{\Delta V_{\text{L, max}}}{\omega L} = \frac{0.20 \text{ V}}{2\pi \left( 50 \times 10^3 \text{ Hz} \right) \left( 50 \times 10^{-6} \text{ H} \right)} = 1.27 \times 10^{-3} \text{ A} \]

33.67. **Model:** Assume any resistance is negligible.

**Visualize:** The potential difference across the inductor and capacitor oscillate.

**Solve:** (a) The current is \( I(t) = I_0 \cos \omega t = (0.50 \text{ A}) \cos \omega t \). Looking at Figure 33.45, we see that the capacitor is fully charged one-quarter cycle after the current is a maximum (or minimum), so the time needed is one quarter of a cycle. We have

\[ \omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left( 20 \times 10^{-3} \text{ H} \right) \left( 8.0 \times 10^{-6} \text{ F} \right)}} = 2.5 \times 10^3 \text{ rad} / \text{s} \]

\[ \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{2.5 \times 10^3 \text{ rad} / \text{s}} = 2.51 \times 10^{-3} \text{ s} = 2.51 \text{ ms} \Rightarrow \Delta t = \frac{1}{4} T = 0.628 \text{ ms} \]

(b) The maximum inductor current and maximum capacitor charge are related by \( I_0 = \omega Q_0 \). The potential across the capacitor is

\[ \Delta V_{\text{C}} = \frac{Q_0}{C} = \frac{I_0}{\omega C} = \frac{0.50 \text{ A}}{\left( 2.5 \times 10^3 \text{ rad} / \text{s} \right) \left( 8.0 \times 10^{-6} \text{ F} \right)} = 25 \text{ V} \]

33.68. **Model:** Assume any resistance is negligible.

**Visualize:** The potential difference across the inductor and capacitor oscillate. The period of oscillation depends on the inductance and capacitance.

**Solve:** The current is \( I(t) = I_0 \cos \omega t = (0.60 \text{ A}) \cos \omega t \). We can relate the extreme values of the current and the capacitor potential difference. Using \( \omega = \frac{1}{\sqrt{LC}} \) and \( Q = C \Delta V_{\text{C}} \), we find

\[ I_0 = \omega Q_0 = \omega C \Delta V_{\text{C, max}} = \frac{1}{\sqrt{LC}} C \Delta V_{\text{C, max}} \Rightarrow C = \frac{I_0^2 L}{\Delta V_{\text{C, max}}} = \frac{(0.60 \text{ A})^2 \left( 10 \times 10^{-3} \text{ H} \right)}{(60 \text{ V})^2} = 1.0 \mu\text{F} \]

**Assess:** This is a reasonable capacitance.

33.69. **Model:** Assume any resistance is negligible.

**Visualize:** Energy in the capacitor and inductor oscillates as the charge and current oscillate, but the total energy is conserved.

**Solve:** The current through the inductor is zero when the charge on the capacitor is maximum. Thus the total energy is stored in the capacitor:

\[ U_{\text{total}} = U_{\text{c}} = \frac{1}{2} \frac{Q_0^2}{C^2} \]

At a later time, when the capacitor’s energy equals the inductor’s energy, they each have half the total energy. Thus

\[ U_{\text{c}} = U_{\text{l}} = \frac{1}{2} U_{\text{total}} \Rightarrow \frac{1}{2} \frac{Q}{C^2} = \frac{1}{2} (\frac{1}{2} \frac{Q_0^2}{C^2}) \Rightarrow Q = \frac{Q_0}{\sqrt{2}} = 0.707 Q_0 \]

33.70. **Visualize:** The oscillation frequency depends on the capacitance and the inductance. The inductance will depend on the number of turns, the length, and the cross-sectional area of the solenoid.

**Solve:** First we’ll find the inductance necessary to give the proper frequency, then we can design our inductor.

For an LC circuit,

\[ \omega = \frac{1}{\sqrt{LC}} = 2\pi f \Rightarrow L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2 (1.0 \text{ Hz})^2 (1.0 \text{ F})} = 2.53 \times 10^{-2} \text{ H} \]
The inductor length is \( l = Nd \), where \( N \) is the number of turns and \( d \) is the diameter of the wire used. We find that

\[
L = \frac{\mu_0 N^2 A}{l} = \frac{\mu_0 N^2 \pi r^2}{Nd} \Rightarrow N = \frac{dL}{\mu_0 \pi r^2} = \frac{(0.25 \times 10^{-3} \text{ m})(2.53 \times 10^{-3} \text{ H})}{(4\pi \times 10^{-7} \text{ T m / A})\pi(0.02 \text{ m})^2} = 4000
\]

If we have one layer, the length will be \( l = Nd = 4000(0.25 \times 10^{-3} \text{ m}) = 1.0 \text{ m} \). Our inductor can be 1.0 m long with 4000 turns of wire. Alternatively, a 0.5-m-long coil with two layers of 2000 turns would also work.

33.71. **Model:** Assume negligible resistance in the LC part of the circuit.

**Visualize:** With the switch in position 1 for a long time the capacitor is fully charged. After moving the switch to position 2, there will be oscillations in the LC part of the circuit.

**Solve:** (a) After a long time the potential across the capacitor will be that of the battery and \( Q_0 = C\Delta V_{\text{bat}} \). When the switch is moved, the capacitor will discharge through the inductor and LC oscillations will begin. The maximum current is

\[
I_0 = \omega Q_0 = \omega C\Delta V_{\text{bat}} = \frac{C\Delta V_{\text{bat}}}{\sqrt{LC}} = \frac{C}{\sqrt{L}} \Delta V_{\text{bat}} = \frac{\sqrt{2.0 \times 10^{-4} \text{ F}}}{\sqrt{(50 \times 10^{-3} \text{ H})(12 \text{ V})}} = 7.6 \times 10^{-2} \text{ A} = 76 \text{ mA}
\]

(b) The current will be a maximum one-quarter cycle after the maximum charge. The period is

\[
T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC} = 2\pi\sqrt{(50 \times 10^{-3} \text{ H})(2.0 \times 10^{-5} \text{ F})} = 2.0 \text{ ms}
\]

So the current is first maximum at \( t_{\text{max}} = \frac{1}{4}T = 0.50 \text{ ms} \).

33.72. **Model:** Assume negligible resistance in the circuit.

**Visualize:** Energy is conserved. The maximum voltage on the right capacitor will occur when all of the energy from the left capacitor is transferred to the right one.

**Solve:** (a) The voltage is calculated as follows:

\[
\frac{1}{2} C_1 \Delta V_{c1}^2 = \frac{1}{2} C_2 \Delta V_{c2}^2 \Rightarrow \Delta V_{c1} = \sqrt{\frac{C_1}{C_2}} \Delta V_{c2} = \sqrt{\frac{300}{1200}(100 \text{ V})} = 50 \text{ V}
\]

(b) Closing S1 causes the charge and current of the left LC circuit to oscillate with period \( T_1 \). After one-quarter of a period, the 300 \( \mu \text{F} \) capacitor is completely discharged and the current through the inductor is maximum. At that instant we’ll open S1 and close S2. Then the right LC circuit will start to oscillate with period \( T_2 \) and the inductor current will charge the 1200 \( \mu \text{F} \) capacitor. The capacitor will be fully charged after one-quarter of a period, so we will open S2 at that time to keep the charge on the 1200 \( \mu \text{F} \) capacitor. The periods are

\[
T_1 = \frac{2\pi}{\omega_1} = 2\pi\sqrt{LC_1} = 2\pi\sqrt{(5.3 \text{ H})(300 \times 10^{-6} \text{ F})} = 0.25 \text{ s} \Rightarrow \frac{1}{4}T_1 = 0.0625 \text{ s}
\]

\[
T_2 = \frac{2\pi}{\omega_2} = 2\pi\sqrt{LC_2} = 2\pi\sqrt{(5.3 \text{ H})(1200 \times 10^{-6} \text{ F})} = 0.50 \text{ s} \Rightarrow \frac{1}{4}T_2 = 0.1250 \text{ s}
\]

So the procedure is to close S1 at \( t = 0 \text{ s} \), open S1, and close S2 at \( t = 0.0625 \text{ s} \), then open S2 at \( t = 0/1250 \text{ s} \).

33.73. **Model:** Assume an ideal inductor and an ideal (resistanceless) battery.

**Visualize:** Please refer to figure P33.73. The current through the battery is the sum of the currents through the left and right branches of the circuit.

**Solve:** (a) Because the switch has been open a long time, no current is flowing the instant before the switch is closed. A basic property of an ideal inductor is that the current through it cannot change instantaneously. This is because the potential difference \( \Delta V_i = -L(dI/dt) \) would become infinite for an instantaneous change of current, and that is not physically possible. Because the current through the inductor was zero before the switch was closed, it must still be zero (or very close to it) immediately after the switch is closed. Conservation of current requires the current through the entire right branch to be the same as that through the inductor, so it is also zero immediately after the switch is closed. The only current is through the left 10 \( \Omega \) resistor, which sees the full battery potential of the battery. Thus \( I_{\text{bat}} = I_{\text{bat}} = \Delta V_{i0}/R = (10 \text{ V})/(10 \Omega) = 1.0 \text{ A} \).

(b) The current through the inductor increases as time passes. Once the current \( I_{\text{bat}} \) reaches a steady value and is no longer changing, the potential difference across the inductor is \( \Delta V_i = -L(dI/dt) = 0 \text{ V} \). An ideal inductor has no resistance, so the inductor simply acts like a wire and has no effect on the circuit. The circuit is that of two 10 \( \Omega \) resistors in parallel. The equivalent resistance is 5 \( \Omega \) and the battery current is \( I_{\text{bat}} = (10 \text{ V})/(5 \Omega) = 2.0 \text{ A} \).
33.74. **Model:** Assume an ideal inductor and an ideal (resistanceless) battery.

**Visualize:** Please refer to figure P33.74.

**Solve:**
(a) Because the switch has been open a long time, no current is flowing the instant before the switch is closed. A basic property of an ideal inductor is that the current through it cannot change instantaneously. This is because the potential difference \( \Delta V_L = -L(dI/dt) \) would become infinite for an instantaneous change of current, and that is not physically possible. Because the current through the inductor was zero before the switch was closed, it must still be zero (or very close to it) immediately after the switch is closed. Consequently, the inductor has no effect on the circuit. It is simply a 10 \( \Omega \) resistor and 20 \( \Omega \) resistor in series with the battery. The equivalent resistance is 30 \( \Omega \), so the current through the circuit (including through the 20 \( \Omega \) resistor) is 
\[ I = \frac{\Delta V_{bat}}{R_{eq}} = \frac{(30 \text{V})/(30 \Omega)}{30 \Omega} = 1.0 \text{A}. \]

(b) After a long time, the currents in the circuit will reach steady values and no longer change. With steady currents, the potential difference across the inductor is \( \Delta V_L = -L(dI/dt) = 0 \text{V} \). An ideal inductor has no resistance \( (R = 0 \Omega) \), so the inductor simply acts like a wire. In this case, the inductor "shorts out" the 20 \( \Omega \) resistor. All current from the 10 \( \Omega \) resistor flows through the resistanceless inductor, so the current through the 20 \( \Omega \) resistor is 0 A.

(c) When the switch has been closed a long time, and the inductor is shorting out the 20 \( \Omega \) resistor, the current passing through the 10 \( \Omega \) resistor and through the inductor is \( I = (30 \text{V})/(10 \Omega) = 3.0 \text{A} \). Because the current through an inductor cannot change instantaneously, the current must remain 3.0 A immediately after the switch reopens. This current must go somewhere (conservation of current), but now the open switch prevents the current from going back to the battery. Instead, it must flow upward through the 20 \( \Omega \) resistor. That is, the current flows around the LR circuit consisting of the 20 \( \Omega \) resistor and the inductor. This current will decay with time, with time constant \( \tau = LR \), but immediately after the switch reopens the current is 3.0 A.

33.75. **Visualize:** Please refer to Figure P33.75.

**Solve:**
(a) After a long time has passed the current will no longer be changing. With steady currents, the potential difference across the inductor is \( \Delta V_L = -L(dI/dt) = 0 \text{V} \). An ideal inductor has no resistance \( (R = 0 \Omega) \), so the inductor simply acts like a wire. The circuit is simply that of a battery and resistor \( R \), so the current is
\[ I_o = \frac{\Delta V_{bat}}{R} = \frac{R}{L} \left( I_0 - I \right) \]

(b) In general, we need to apply Kirchhoff’s loop law to the circuit. Starting with the battery and going clockwise, the loop law is
\[ \Delta V_{bat} + \Delta V_R + \Delta V_L = \Delta V_{bat} - IR - L \frac{dI}{dt} = 0 \]

This is a differential equation that we can solve by direct integration. The current is \( I = 0 \text{A} \) at \( t = 0 \text{s} \), so separate the current and time variables and then integrate from 0 A at 0 s to current \( I \) at time \( t \):

\[ \int_0^t \frac{dl}{I_0 - I} = \frac{R}{L} \int_0^t dt \Rightarrow \ln((I_0 - I)_{I_0}) = \frac{R}{L} t \Rightarrow (I_0 - I)_{I_0} = e^{\frac{R}{L} t} \]

Taking the exponential of both sides gives
\[ \frac{I_o - I}{I_o} = e^{-\frac{R}{L} t} \Rightarrow I_o - I = I_o e^{-\frac{R}{L} t} \]

Finally, solving for \( I \) gives
\[ I = I_o (1 - e^{-\frac{R}{L} t}) \]

The current is 0 A at \( t = 0 \text{s} \), as expected, and exponentially approaches \( I_o \) as \( t \to \infty \).

(c) **Assess:** The current is zero at the start and approaches the steady final value. The behavior is similar to the charging of a capacitor.
33.76. Visualize: Please refer to Figure CP33.76. The area within in the loop is changing so the flux will change. This will produce an induced emf and corresponding current. The area of the loop is the area of a square.
Solve: (a) The field is out of the page, so the flux is increasing outward as the loop expands. According to Lenz’s law, the induced current tries to prevent the increase of outward flux. It does so by generating a field into the page. This requires a clockwise current flow.

(b) Take \( A \) parallel to \( \vec{B} \), so \( \Phi = AB \) and \( \dot{\vec{E}} = \frac{d\Phi}{dt} = B(dA/dt) \). The field is constant but the loop area is changing. Let the length of each edge of the loop be \( x \) at time \( t \). This length is increasing linearly with time as the corner of the loop moves outward with speed \( v = 10 \text{ m/s} \). We have

\[
x = x_0 + vt = 0 + [(10 \text{ m/s})\cos 45^\circ]t = 7.07t \text{ m}
\]

The loop’s area at time \( t \) is

\[
A = x^2 = (7.07t \text{ m})^2 = 50t^2 \text{ m}^2 \Rightarrow \frac{dA}{dt} = 100t \text{ m}^2/\text{s}
\]

Consequently, the induced emf at time \( t \) is

\[
\mathcal{E} = B\frac{dA}{dt} = (0.1 \text{ T})(100t \text{ m}^2/\text{s}) = 10t \text{ V}
\]

The induced current is \( I = \mathcal{E}/R \), where \( R \) is the resistance of the loop. The wire has resistance 0.01 \( \Omega \) per meter, so

\[
R = 4(0.01 \text{ \Omega/m})(7.07t \text{ m}) = 0.282t \text{ \Omega}
\]

Thus, the induced current is

\[
I = \frac{\mathcal{E}}{R} = \frac{10t \text{ V}}{0.282t \text{ \Omega}} = 35.5 \text{ A}
\]

(c) At \( t = 0.1 \text{ s} \), \( \mathcal{E} = 1.0 \text{ V} \) and \( I = 35.5 \text{ A} \).
Assess: The induced emf depends on time, but so does the resistance. This means the induced current is constant.

33.77. Model: Assume the straight wire is very long so we can use the formula for the magnetic field produced by an “infinite” wire.
Visualize: Please refer to Figure CP33.77. As the short wire moves in the field of the long wire, the charges in the short wire will experience a magnetic force. They will move inside the wire until the electric field created by the charge separation has a corresponding force that balances the magnetic force. The potential difference between the ends of the wire is due to this electric field.
Solve: To find the potential difference between the ends of the short wire we need to know the electric field along it. The magnetic field of the long wire is into the page near the short wire, so the magnetic force on positive charges will be to the left. Positive charges will move to the left, leaving negative charges on the right. This creates an electric field and corresponding force to the right on the positive charges. Consider the charges at a distance \( x \) from the long wire. In equilibrium, the magnetic and electric forces balance. Using the magnetic field for an “infinite” wire,

\[
qvB = q\vec{E} \Rightarrow E = vB = \frac{\mu_0 I}{2\pi x}
\]

Integrating the electric field from the negative end of the wire (right end) to the positive end (left end), the potential difference is

\[
\Delta V = -\int_{x^i}^{x^f} \vec{E} \cdot d\vec{l} = \int_{x^i}^{x^f} \left(\frac{v\mu_0 I}{2\pi x}\right) \hat{x} \cdot d\hat{x} = \left(\frac{v\mu_0 I}{2\pi}\right) \int_{x^i}^{x^f} \frac{dx}{x} = \left(\frac{v\mu_0 I}{2\pi}\right) \ln\left(\frac{d + l}{d}\right)
\]

where we have used \( d\hat{l} = -dx\hat{x} \).
Assess: This seems reasonable, because the potential difference increases with current (stronger magnetic field) and velocity (larger motional emf).

33.78. Model: Assume that the sides remain straight during the collapse. Also, assume the magnetic field is constant in the region of interest.
Visualize: As the loop collapses the area will change, there will be a corresponding change in flux and, therefore, an induced emf and current. The loop is made up of four right triangles of equal size with width \( x \), height \( y \), and hypotenuse \( b = 10 \text{ cm} \). The corner of the loop is moving in the \( x \) direction.
Solve: (a) For simplicity we will focus on the upper-right triangle. The width of the upper-right triangle is given by \( x = x_0 + vt \), with \( x_0 = \frac{b}{\sqrt{2}} = (0.10 \text{ m})/\sqrt{2} = 0.0707 \text{ m} \) and \( v = 0.293 \text{ m/s} \). The height \( y \) of the triangle is \( y = \sqrt{b^2 - x^2} \). The loop will be a straight line when \( x = b \). The time when this occurs is \( t_{\text{collapse}} = \frac{b - x_0}{v} = \frac{0.10 \text{ m} - 0.0707 \text{ m}}{0.293 \text{ m/s}} = 0.100 \text{ s} \).

(b) The induced current is due to the changing flux as a result of the changing area. We have

\[
I = \frac{E}{R} = \frac{1}{R} \frac{d\Phi}{dt} = \frac{B}{R} \frac{dA}{dt}
\]

The total area at any time is four times the area of the upper-right triangle:

\[
A = 4 \times \frac{1}{2} xy = 2x\sqrt{b^2 - x^2}
\]

Using the chain rule,

\[
I = \frac{B}{R} \frac{dA}{dx} \frac{dx}{dt} = \frac{B}{R} \frac{d}{dx} \left( 2x\sqrt{b^2 - x^2} \right) \frac{dx}{dt} = \frac{2Bv}{R} \sqrt{b^2 - x^2}
\]

\[
= \frac{2(0.293 \text{ m/s})(0.50 \text{ T})}{0.10 \Omega} \left( \frac{(0.10 \text{ m})^2 - 2[0.0707 \text{ m} + (0.293 \text{ m/s})]t}{\sqrt{(0.10 \text{ m})^2 - [0.0707 \text{ m} + (0.293 \text{ m/s})]^2}} \right)
\]

\[
= 2.93 \left( \frac{(0.10)^2 - 2[0.0707 + (0.293)t]^2}{\sqrt{(0.10)^2 - [0.0707 + (0.293)t]^2}} \right) A
\]

where \( t \) is in seconds.

(c) \[
\begin{array}{c|c}
 t (s) & I (A) \\
 0.000 & 0.000 \\
 0.020 & 0.078 \\
 0.040 & 0.186 \\
 0.060 & 0.348 \\
 0.080 & 0.671 \\
 0.090 & 1.078 \\
 0.100 & \text{diverges}
\end{array}
\]
**33.79. Model:** Assume the field changes abruptly at the boundary and is constant.

**Visualize:**

The loop moving through the field will have a changing flux. This will produce an induced emf and corresponding current. The induced current will oppose the change in flux and result in a retarding force on the loop.

**Solve:**

(a) The loop moving into the field will have an increasing outward flux. The induced current will oppose this change and create a field that is into the page. This requires that the induced current flow clockwise. The current-carrying wires of the loop will experience a force. The force on the leading edge is to the left, retarding the motion. The forces on the top and bottom edges cancel and the trailing edge experiences no force since the field is zero there. The induced current is

\[
I = \frac{\mathcal{E}}{R} = \frac{1}{R} \frac{d\Phi}{dt} = \frac{B}{R} \frac{dA}{dt} = \frac{Bl}{R} \frac{dx}{dt} = \frac{Blv}{R}
\]

With this current we can find the force on the leading edge. Remembering that it opposes the motion, we have

\[
F = liB = \left(\frac{Blv}{R}\right)B = \left(\frac{B^2l^2}{R}\right)v
\]

Newton’s second law is

\[
ma = m \frac{dv}{dt} = -\left(\frac{B^2l^2}{R}\right)v \Rightarrow \frac{dv}{dt} = -\left(\frac{B^2l^2}{mR}\right)v \Rightarrow v = v_0 e^{-bt}
\]

where \( b = \frac{l^2B^2}{mR} \) and the initial velocity is \( v_0 \).

(b) The value of \( b \) is

\[
b = \frac{(0.1 \text{ m})^2(0.1 \text{ T})^2}{(0.001 \text{ kg})(0.001 \Omega)} = 100 \text{ s}^{-1}
\]

**Assess:** This exponential decay of the velocity is called eddy-current braking.
33.80. **Model:** Assume the field changes uniformly when turned on and that the loop is initially at rest.

**Visualize:**

The changing magnetic field produces a changing flux that will generate an induced emf and current. The current-carrying wire will experience a force that opposes the change.

**Solve:**

(a) The increasing field strength produces an increasing outward flux. The induced current will oppose this change and create a field that is into the page. This requires that the induced current flow clockwise. The current-carrying wires of the loop will experience a force. Take \( A \parallel B \) so the flux is \( \Phi = A \cdot B = AB \).

Because the field strength is changing and the loop is moving, the emf has two terms. We have

\[
E = \frac{d\Phi}{dt} = \frac{1}{R} \left( A \frac{dB}{dt} + \frac{dA}{dt} B \right)
\]

We will assume that the loop does not move much and ignore the \( dA/dt \) term (see below). The current is

\[
I = \frac{E}{R} = \frac{1}{R} A \frac{dB}{dt} = \frac{I^2 \Delta B}{2R \Delta t} = \frac{(0.080 \text{ m})^2 (100 \text{ T/s})}{2(0.010 \Omega)} = 32 \text{ A}
\]

(b) For a clockwise current, the force on the leading edge is to the left, so it is away from the magnetic field. The forces on the top and bottom edges cancel, and the trailing edge experiences no force since the field is zero there. The force on the loop is

\[
F = IIB = (32 \text{ A})(0.080 \text{ m})(0.50 \text{ T}) = 1.28 \text{ N}
\]

(c) Using Newton's second law,

\[
a = \frac{F}{m} = \frac{1.28 \text{ N}}{0.010 \text{ kg}} = 128 \text{ m/s}^2
\]

If the acceleration is constant, the distance the loop moves is

\[
\Delta s = \frac{1}{2} a t^2 = \frac{1}{2} (128 \text{ m/s}^2)(0.010 \text{ s})^2 = 6.4 \times 10^{-3} \text{ m} = 0.64 \text{ cm}
\]

(d) The distance traveled is about a factor of ten smaller than the size of the loop, so ignoring the motion of the loop is reasonable.

(e) To find the speed after being "kicked" we calculate the impulse. Using the average force found in the part (b),

\[
F_{av} \Delta t = \Delta p = mv_f - mv_i = \frac{F_{av} \Delta t}{m} = \frac{(1.28 \text{ N})(0.010 \text{ s})}{0.010 \text{ kg}} = 1.28 \text{ m/s}
\]

**Assess:** This shows that the loop would be shot out of the field region.

33.81. **Model:** Assume the conductor is long enough to use the formula for the magnetic field of an "infinite" wire.

**Visualize:** Please refer to Figure CP33.81. The current in the inner conductor creates a field that circles it in the space between the inner and outer conductor. Magnetic flux exists in the space.

**Solve:**

(a) We will consider a rectangle between the conductors of length \( l \) that is perpendicular to the magnetic field produced by the inner wire. Take \( A \parallel B \) so the flux through a small piece of the rectangle of width \( dr \) is \( d\Phi = B \cdot dA = Bldr \). The total flux is

\[
\Phi = \int d\Phi = \int_{r_i}^{r_f} \left( \frac{\mu_0 I}{2\pi} \right) dr = \frac{\mu_0 I}{2\pi} \ln \left( \frac{r_f}{r_i} \right)
\]
where we have used the formula for the field of an infinite wire. We want the inductance per unit length, so we take the inductance and divide by the length. We have

$$L = \frac{\Phi}{I} \Rightarrow \bar{L} = \frac{\Phi}{\mu I} = \frac{\mu_0}{2\pi} \ln\left(\frac{r_2}{r_1}\right).$$

(b) The inductance per unit length is

$$\bar{L} = \frac{\left(4\pi \times 10^{-7} \text{ T m / A}\right)}{2\pi} \ln\left(\frac{3.0}{0.50}\right) = 3.6 \times 10^{-7} \text{ H / m} = 0.36 \mu\text{H / m}.$$